EPFL

Semaine 14
QCMs

ME-332 – Mécanique Vibratoire

École
polytechnique
fédérale
de Lausanne

Plan du cours

Week	Jours	Partie du cours	Exercices	Homework
1	09.09//11.09	Intro et oscillateur 1D libre conservatif	Série 1	Leçon 1.1&1.2
2	18.09	Oscillateur 1D libre dissipatif	Série 2	Leçon 2.1&2.2
3	23.09//25.09	Régime permanent harmonique (complexe)	Série 3	Leçon 3.1&3.2
4	30.09//02.10	Régime permanent périodique (série de Fourier)	Série 4	Leçon 4.1&4.2
5	07.10//09.10	Régime forcé général (transf. Laplace ou Fourier)	Série 5	Leçon 5.1&5.2
6	14.10//16.10	Systèmes à 2 DDL conservatifs	Série 6	Leçon 6.1&6.2
BREAK!!!				
7	28.10//30.10	Récapitulatif // séance de QCMs	MIDTERM	
8	04.11//06.11	Systèmes à 2DDL dissipatifs, osc. de Frahm	Série 7	Leçon 7.1&7.2
9	11.11//13.11	Systèmes à n DDL, conservatif	Série 8a	Leçon 8.1&8.2
10	18.11//20.11	Systèmes à n DDL, conservatif	Série 8b	
11	25.11//27.11	Systèmes à n DDL, dissipatif	Série 9	Leçon 9.1&9.2
12	02.12//04.12	Systèmes à n DDL, forcé	Série 10	Leçon 10.1&10.2
13	09.12//11.12	Systèmes continus: barres & poutres	Série 11	Leçon 11.1&11.2
14	16.12//18.12	Récapitulatif // séances de QCMs		

Before we start... let's connect!

- Smartphones
  - iPhone
  - Android

Download and install the app "*PointSolutions*"

- Computers (or Windows Phones)
  - Go to www.responseware.eu
- Choose **EUROPE Server** (if asked)
- Join session – **ME332**
- When asked – **do not input your name. Enter as anonymous.**

Final Exam

- Date: 28 Janvier (2025)
- Heure: 15h30 – 18h00 (2h30)
  - Doors open at 15h15
- Salles: CE16, PO01
- Contenu: Tout le semestre
- Formulaire:
  - Une page A4 «recto-verso»

- Pas oublier:
  - Stylo (Ni rouge ni vert – PAS crayon)
  - Calculatrice (pas matrices)
  - EPFL ID – Camipro

- Exam:
  - 1-2 questions courtes
  - 2-3 problèmes

- 70% de la note finale

- Séance de doutes:
  - Salle: ???
  - Date: 24 Janvier 2024
  - Heure: ???

Si $C = c\mathbb{I}$, C^0 en coordonnées modales?

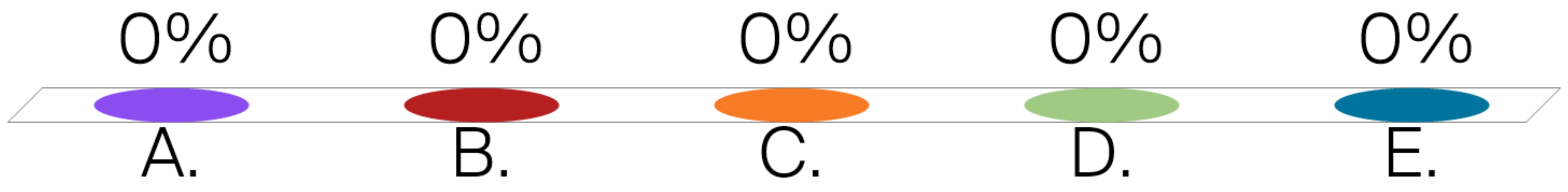
A. $K = k\mathbb{I}$

B. $K = 2k\mathbb{I}$

C. $C^0 = c\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

D. Ça dépend

E. Je ne sais pas



Si $C = c\mathbb{I}$, C^0 en coordonnées modales?

$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$; $\vec{\beta}_{II} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$; $\vec{\beta}_{III} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

A. $K = k\mathbb{I}$

B. $K = 2k\mathbb{I}$

C. $C^0 = c \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

D. Ça dépend

E. Je ne sais pas

$c_{I,eff} = c \cdot |\beta_I|^2$

$c_{II}\ eff = c \cdot |\beta_{II}|^2$

$c_{III,eff} = c\ |\beta_{III}|^2$

$C^0 = B^T \cdot C \cdot B$

$C^0 = c \cdot B^T \cdot B$

M, K, C

$C M^{-1} K = K M^{-1} C$

$C = \alpha \cdot M + \beta \cdot K$

A.	B.	C.	D.	E.
0%	0%	0%	0%	0%

Dans quelles C.I. le système bougera à $\omega_{0,I}$?

$\omega_1 = \omega_0\sqrt{1-\eta^2}$

$$\vec{\beta}_I = \begin{pmatrix}1\\1\\1\end{pmatrix};\ \vec{\beta}_{II} = \begin{pmatrix}1\\0\\-1\end{pmatrix};\ \vec{\beta}_{III} = \begin{pmatrix}1\\-2\\1\end{pmatrix}$$

A. $\vec{x}(0) = \begin{pmatrix}1\\0\\0\end{pmatrix}\mathrm{m},\ \dot{\vec{x}}(0) = \begin{pmatrix}0\\0\\0\end{pmatrix}\frac{\mathrm{m}}{\mathrm{s}};$

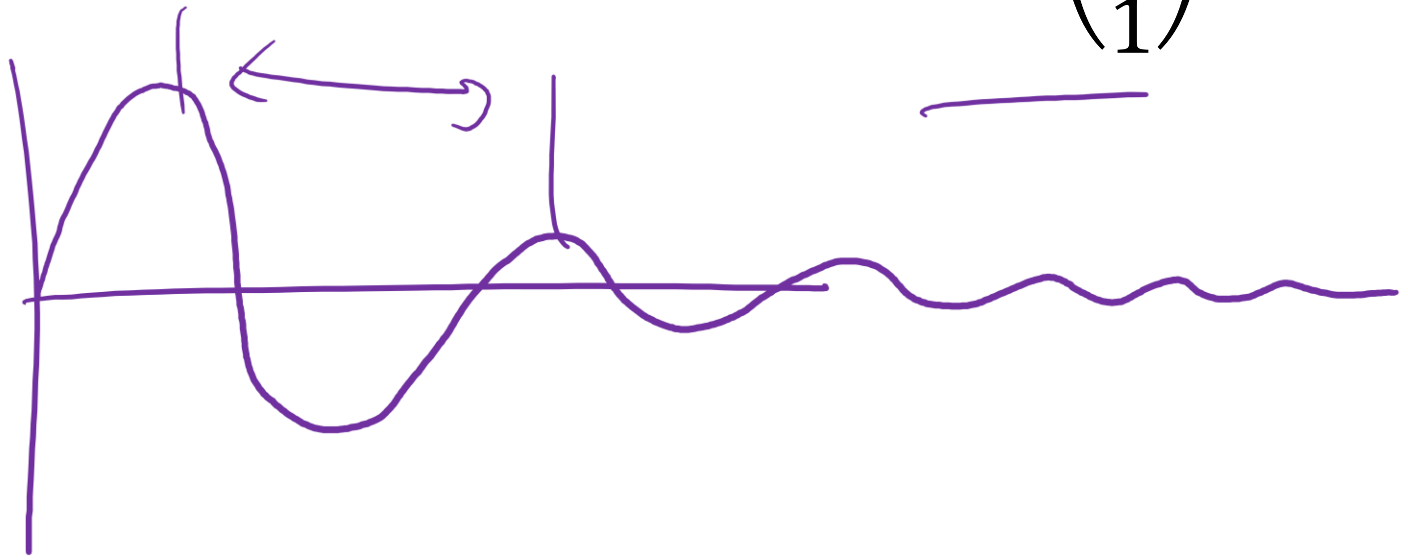
B. $\vec{x}(0) = \begin{pmatrix}1\\1\\1\end{pmatrix}\mathrm{m},\ \dot{\vec{x}}(0) = \begin{pmatrix}0\\0\\0\end{pmatrix}\frac{\mathrm{m}}{\mathrm{s}};$

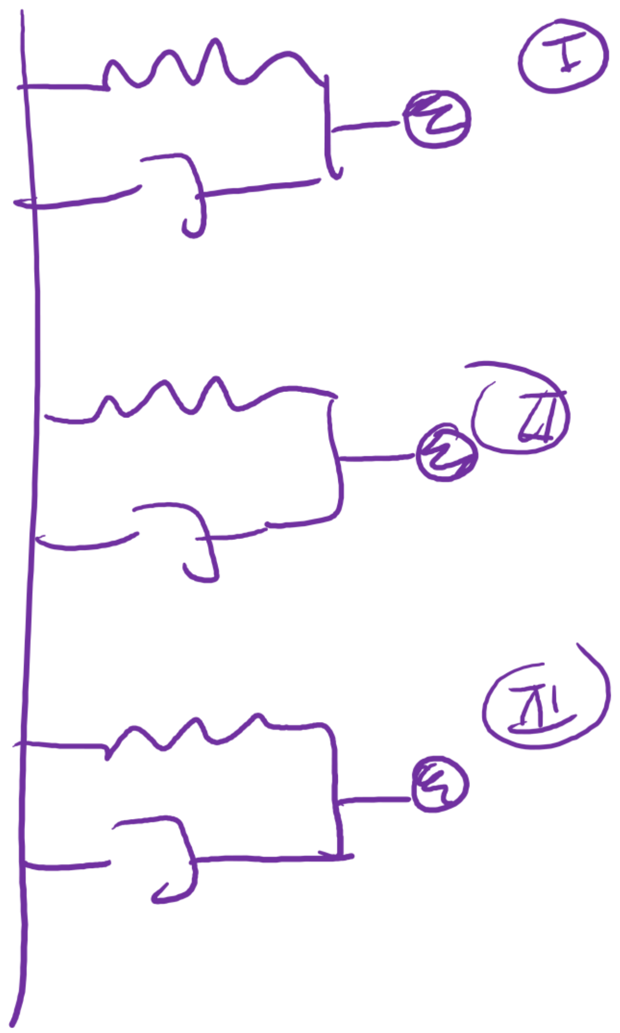
C. $\vec{x}(0) = \begin{pmatrix}0\\0\\0\end{pmatrix}\mathrm{m},\ \dot{\vec{x}}(0) = \begin{pmatrix}1\\1\\1\end{pmatrix}\frac{\mathrm{m}}{\mathrm{s}};$

D. $\vec{x}(0) = \begin{pmatrix}1\\1\\1\end{pmatrix}\mathrm{m},\ \dot{\vec{x}}(0) = \begin{pmatrix}1\\1\\1\end{pmatrix}\frac{\mathrm{m}}{\mathrm{s}};$

E. Je ne sais pas

F. None of the above





0%	0%	0%	0%	0%	0%
A.	B.	C.	D.	E.	F.

Si $\vec{x}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \dot{\vec{x}}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \frac{\mathrm{m}}{\mathrm{s}}$, calculer $\vec{x}(t)$ en régime permanent

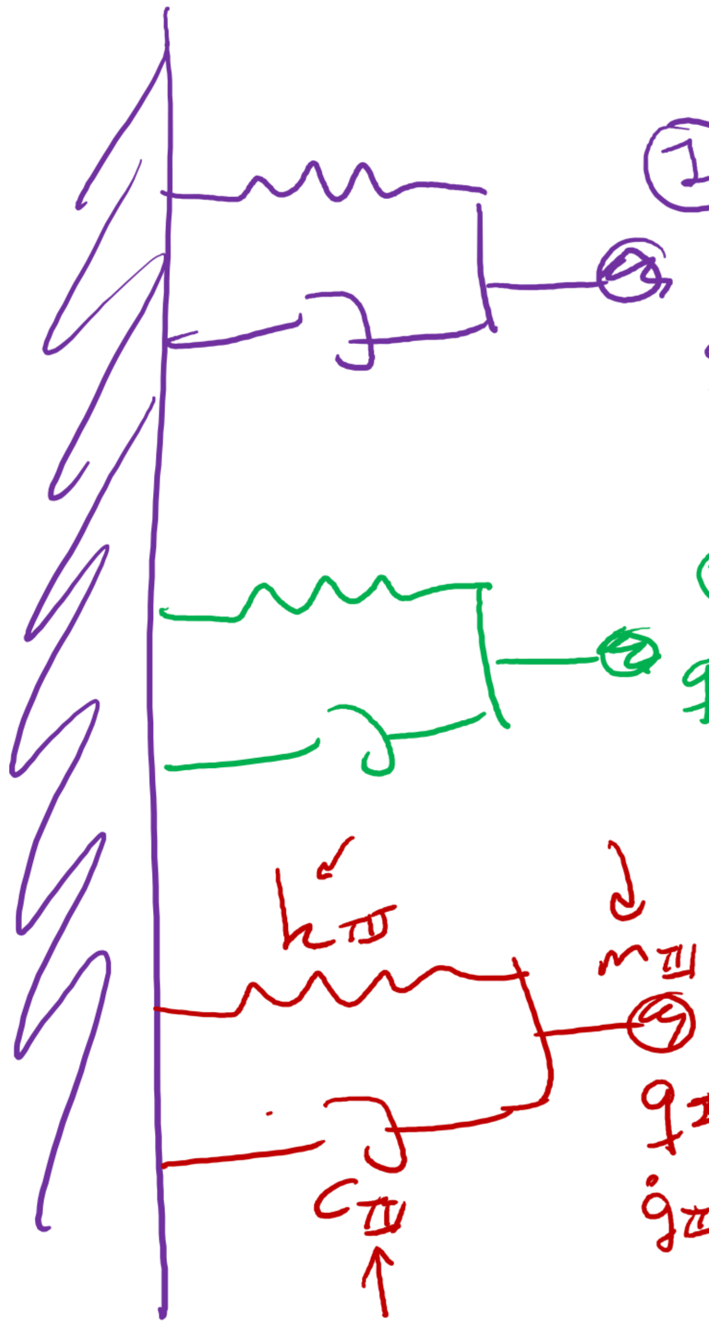
$\vec{x}(t) = \vec{0}$

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix};\ \vec{\beta}_{II} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix};\ \vec{\beta}_{III} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Si $\vec{x}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \dot{\vec{x}}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}}$, calculer $\vec{x}(t)$

$\eta < 1$

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix};\ \vec{\beta}_{II} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix};\ \vec{\beta}_{III} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$



I
$q_I(0) = 0$
$\dot{q}_I(0) = \frac{1}{3}\ m/s$

II
$q_{II}(0) = 0$
$\dot{q}_{II}(0) = \frac{1}{2}\ m/s$

III
$q_{III}(0) = 0$
$\dot{q}_{III}(0) = \frac{1}{6}\ m/s$

$\vec{x}(0) = q_I(0)\cdot\vec{\beta}_I + q_{II}(0)\,\vec{\beta}_{II} + q_{III}(0)\cdot\vec{\beta}_{III}$

$\dot{\vec{x}}(0) = \dot{q}_I(0)\cdot\vec{\beta}_I + \dot{q}_{II}(0)\,\vec{\beta}_{II} + \dot{q}_{III}(0)\,\vec{\beta}_{III}$

$\vec{\beta}_I\cdot\dot{\vec{x}}(0) = \dot{q}_I(0)\,\vec{\beta}_I\cdot\vec{\beta}_I + \cancel{\vec{\beta}_I\,\dot{q}_{II}(0)\,\vec{\beta}_{II}} + \cancel{\vec{\beta}_I\cdot\dot{q}_{III}(0)\cdot\vec{\beta}_{III}}$

$\dot{q}_I(0) = \dfrac{\vec{\beta}_I\cdot\dot{\vec{x}}(0)}{|\beta_I|^2} = \dfrac{1}{3}\ m/s$

$\dot{q}_{II}(0) = \dfrac{\vec{\beta}_{II}\cdot\dot{\vec{x}}(0)}{|\beta_{II}|^2} = \dfrac{1}{2}\ m/s$

$\dot{q}_{III}(0) = \dfrac{\vec{\beta}_{III}\cdot\dot{\vec{x}}(0)}{|\vec{\beta}_{III}|^2} = \dfrac{1}{6}\ m/s$

$q_I(t) = Q_I\,e^{-\lambda_I t}\cdot\cos(\omega_I t - \varphi_I)$

$q_I(0) = Q_I\cdot\cos\varphi_I \Rightarrow \varphi_I = \pi/2$

$q_I(t) = Q_I\cdot e^{-\lambda_I t}\cdot\sin(\omega_I t)$

$\dot{q}_I(t) = Q_I\cdot(-\lambda_I)\cdot e^{-\lambda_I t}\cdot\sin(\omega_I t) + Q_I\cdot e^{-\lambda_I t}\cdot(\omega_I)\cdot\cos(\omega_I t)$

$\dot{q}_I(0) = Q_I\cdot\omega_I = \frac{1}{3}\ m/s \Rightarrow \boxed{Q_I = \frac{1}{3\omega_I}\ m} \Rightarrow \boxed{q_I(t) = \frac{1}{3\omega_I}\,e^{-\lambda_I t}\sin\omega_I t}$

$q_{II}(t) = \dfrac{1}{2\omega_{II}}\cdot e^{-\lambda_{II} t}\cdot\sin(\omega_{II} t)$

$q_{III}(t) = \dfrac{1}{6\omega_{III}}\,e^{-\lambda_{III} t}\cdot\sin(\omega_{III} t)$

$\vec{x} = B\cdot\vec{q} \Rightarrow \vec{q} = B^{-1}\,\vec{x} \Rightarrow \vec{q}(0) = B^{-1}\cdot\vec{x}(0)$

$\dot{\vec{q}}(0) = B^{-1}\cdot\dot{\vec{x}}(0)$

Si $\vec{x}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \dot{\vec{x}}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \frac{\mathrm{m}}{\mathrm{s}}$, calculer $\vec{x}(t)$

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix};\ \vec{\beta}_{II} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix};\ \vec{\beta}_{III} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\boxed{\vec{x}(t) = \underline{\underline{0}} \cdot \vec{q} = q_I(t) \cdot \vec{\beta}_I + q_{II}(t) \cdot \vec{\beta}_{II} + q_{III}(t) \cdot \vec{\beta}_{III}}$$

$$C^0 = c \begin{pmatrix} 3 & & 0 \\ & 2 & \\ 0 & & 6 \end{pmatrix} \rightarrow$$

$c_I = 3c$

$c_{II} = 2c$

$c_{III} = 6c$; $\lambda_{III} = \frac{c_{III}}{2m_{III}}$

$$\omega_{1,III} = \omega_{0,III} \cdot \sqrt{1 - \eta_{III}^2}$$

$$\omega_{0,III} = \sqrt{\frac{k_{III}}{m_{III}}} \qquad \eta_{III} = \frac{\lambda_{III}}{\omega_{0,III}}$$

$$\vec{\beta}_I \cdot \underline{\underline{m}} \cdot \vec{x} = \vec{\beta}_I \underline{\underline{m}} \left(q_I \vec{\beta}_I + q_{II} \cdot \vec{\beta}_{II} + q_{III} \cdot \vec{\beta}_{III} \right)$$

$$\vec{\beta}_i \cdot \underline{\underline{m}} \cdot \vec{\beta}_j \propto \delta_{ij}$$

$$q_I = \frac{\vec{\beta}_I \cdot \underline{\underline{m}} \cdot \vec{x}}{\vec{\beta}_I \cdot \underline{\underline{m}} \cdot \vec{\beta}_I}$$

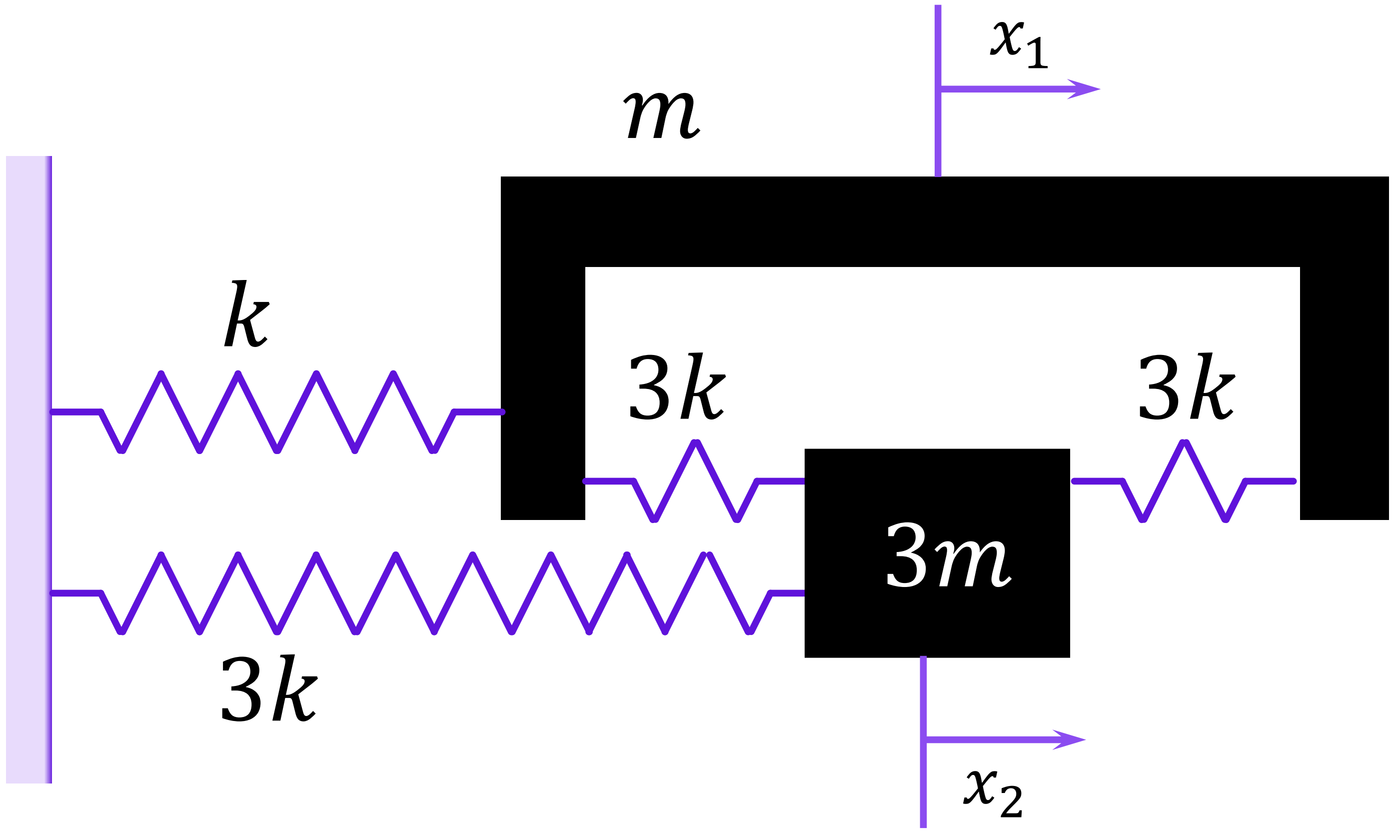
Energie Cinétique?

A. $T = \frac{1}{2} m\dot{x}_1^2 + \frac{1}{2} 3m\dot{x}_2^2$

B. $T = \frac{1}{2} 3m\dot{x}_1^2 + \frac{1}{2} m\dot{x}_2^2$

C. $T = \frac{1}{2} m\dot{x}_1^2 + \frac{1}{2} m\dot{x}_2^2$

D. Je ne sais pas



Energie Potentielle?

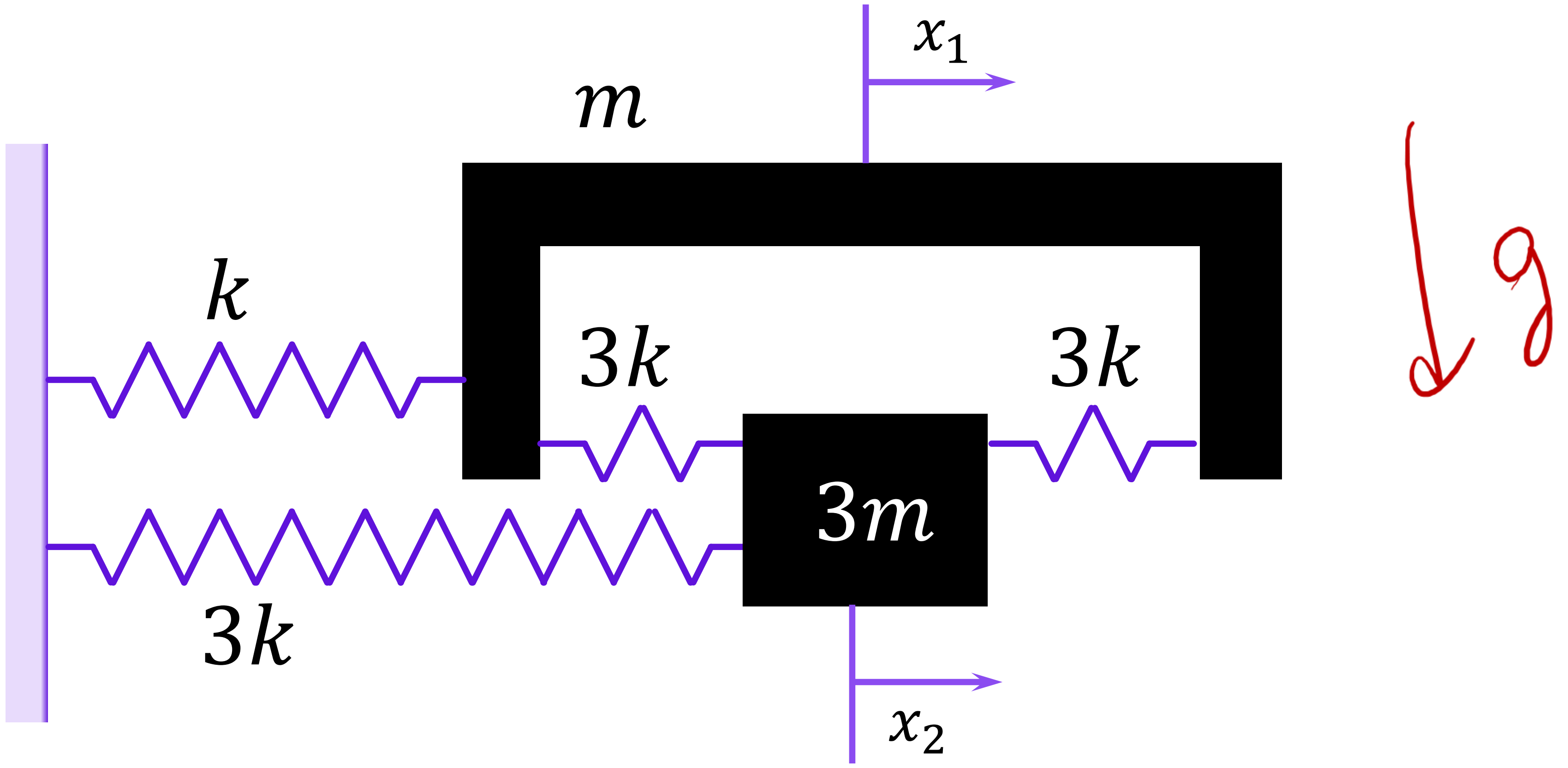
A. $V = \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 + \frac{1}{2}k(x_1 - x_2)^2$

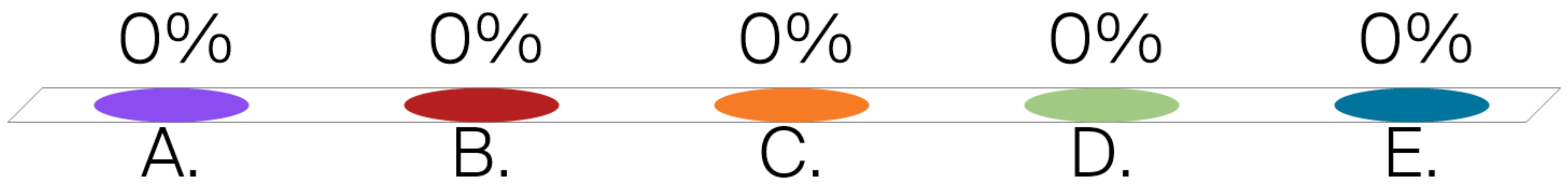
B. $V = \frac{1}{2}kx_1^2 + \frac{3}{2}kx_2^2 + 3k(x_1 - x_2)^2$

C. $V = \frac{1}{2}kx_1^2 + \frac{3}{2}kx_2^2 + 3k(x_1 + x_2)^2$

D. $V = \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 + k(x_1 - x_2)^2$

E. Je ne sais pas





Matrice de masse dans coordonnées x_i?

A. $\begin{pmatrix} m & 0 \\ 0 & 3m \end{pmatrix}$

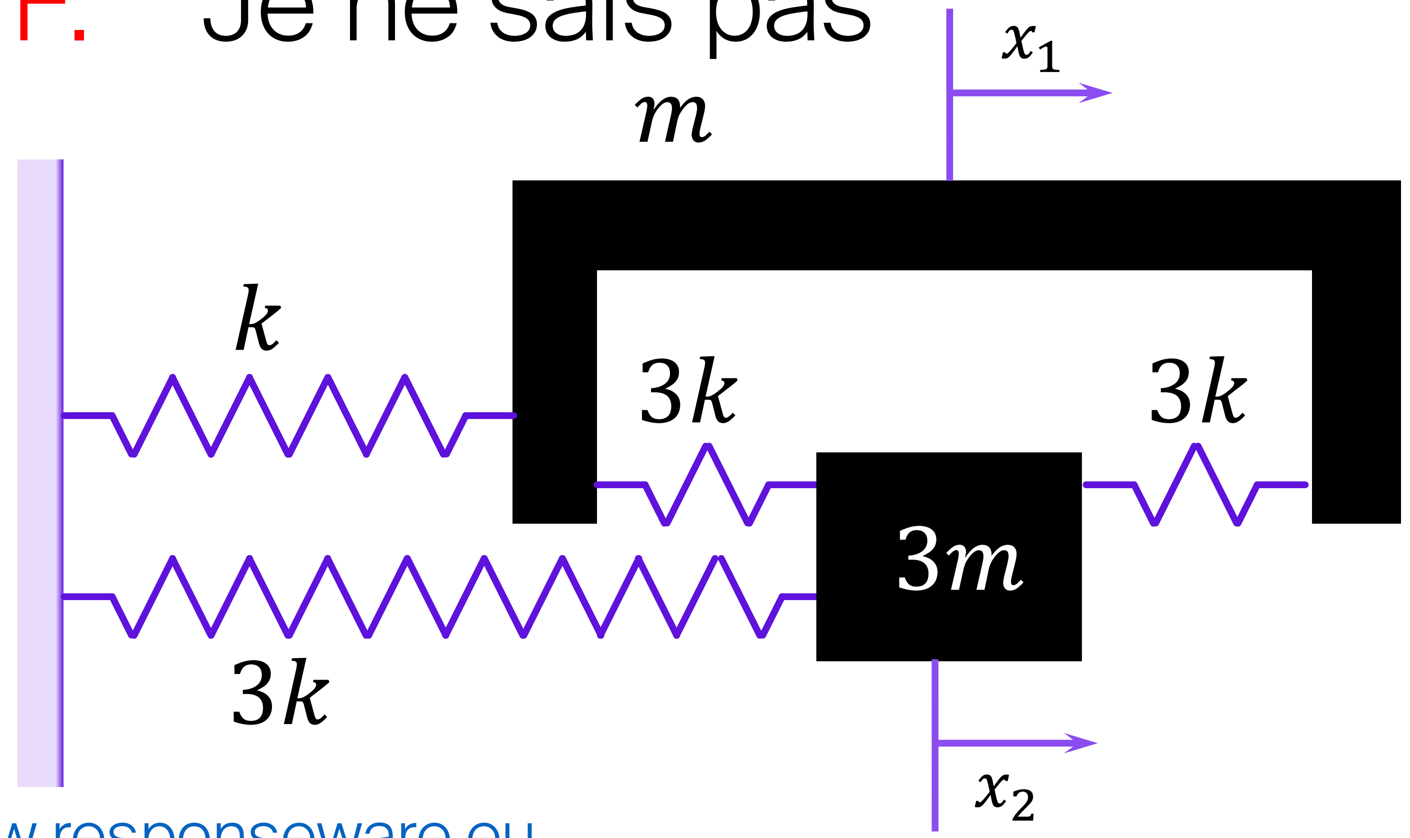
B. $\begin{pmatrix} 3m & 0 \\ 0 & m \end{pmatrix}$

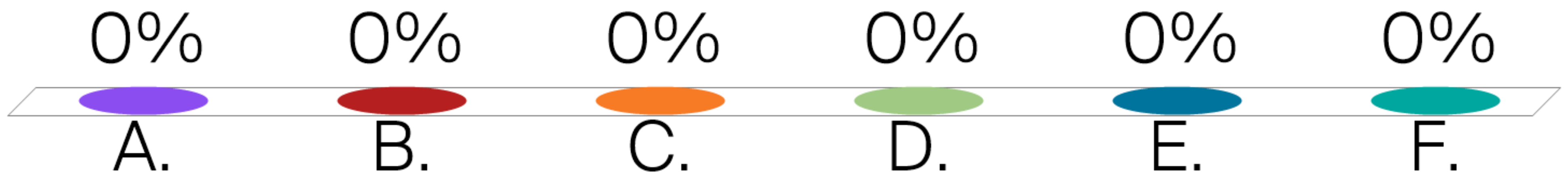
C. $\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$

D. $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

E. Ça dépend

F. Je ne sais pas





Matrice de rigidité dans coordonnées x_i?

$$\boxed{\text{A.} \quad \begin{pmatrix} 7k & -6k \\ -6k & 9k \end{pmatrix}}$$

$$\cancel{\text{B.} \quad \begin{pmatrix} 7 & -6 \\ -6 & 9 \end{pmatrix}}$$

C. $\begin{pmatrix} 7 & 6 \\ 6 & 9 \end{pmatrix} k$

D. $\begin{pmatrix} 7k & 6k \\ 6k & 9k \end{pmatrix}$

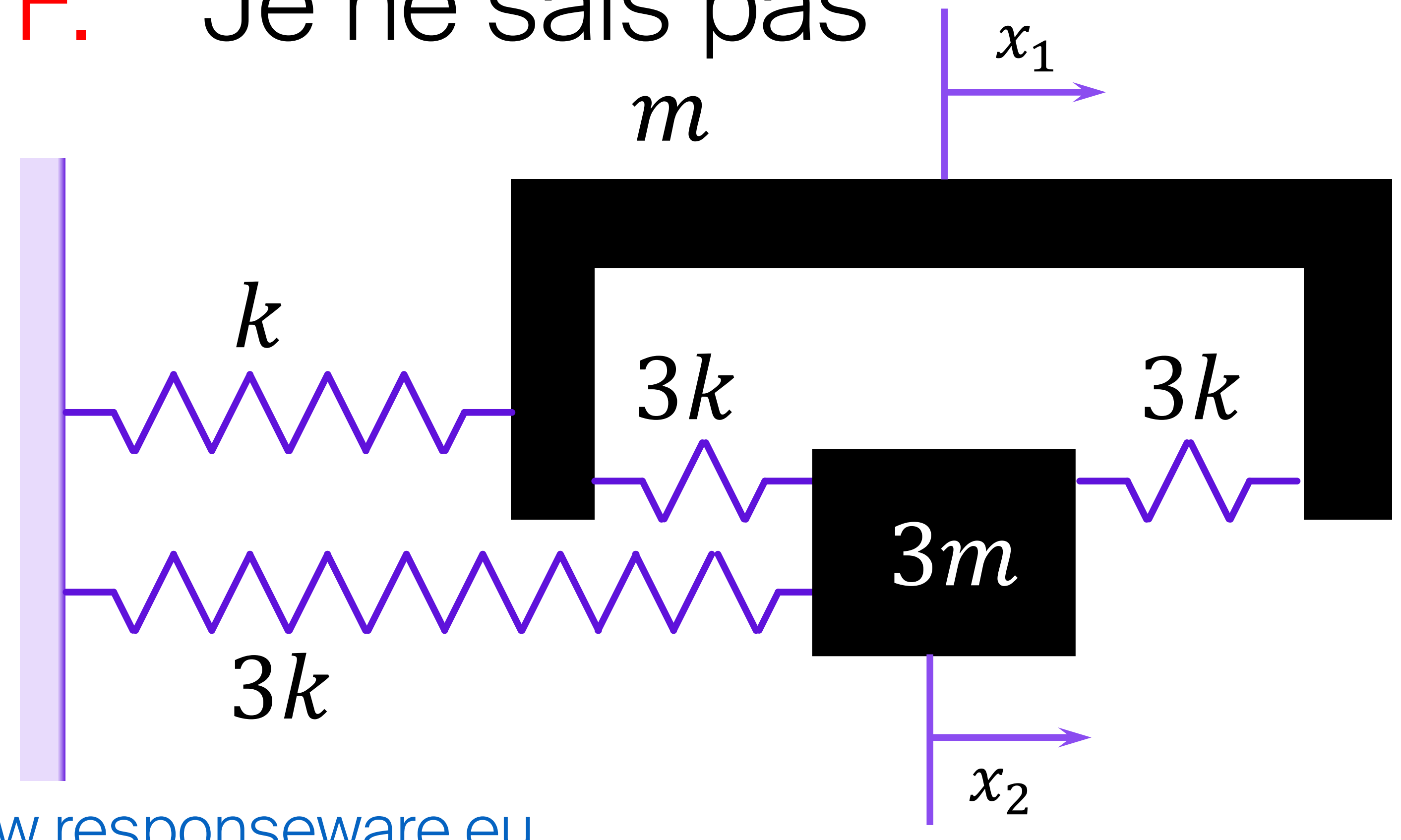
E. Ça dépend

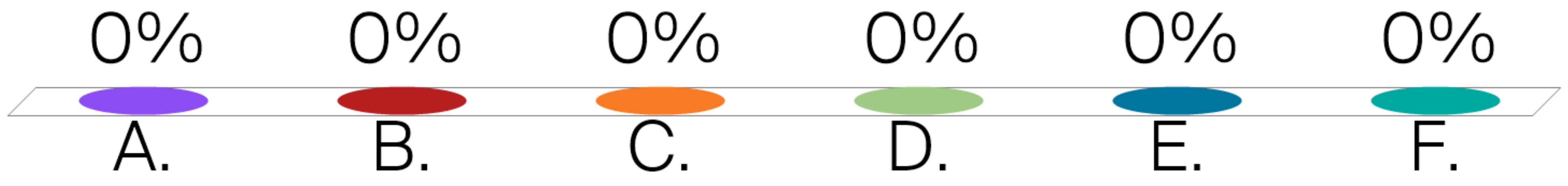
F. Je ne sais pas

$$V = \frac{1}{2} k x_1^2 + \frac{1}{2} 3k x_2^2 + \frac{2}{2} 3k (x_1 - x_2)^2$$

$$\underbrace{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right)} + \underbrace{\frac{\partial V}{\partial x_1}} = 0 \longrightarrow \frac{\partial V}{\partial x_1} = k x_1 + 6k(x_1 - x_2) = \underline{7k} x_1 - \underline{6} k x_2$$

$$\frac{\partial V}{\partial x_2} = 3k x_2 + 6k(x_2 - x_1) = 9 k x_2 - 6 k x_1$$





Matrice de masse dans coordonnées q_i?

A. $\begin{pmatrix} m & 0 \\ 0 & 3m \end{pmatrix}$

B. $\begin{pmatrix} 3m & 0 \\ 0 & m \end{pmatrix}$

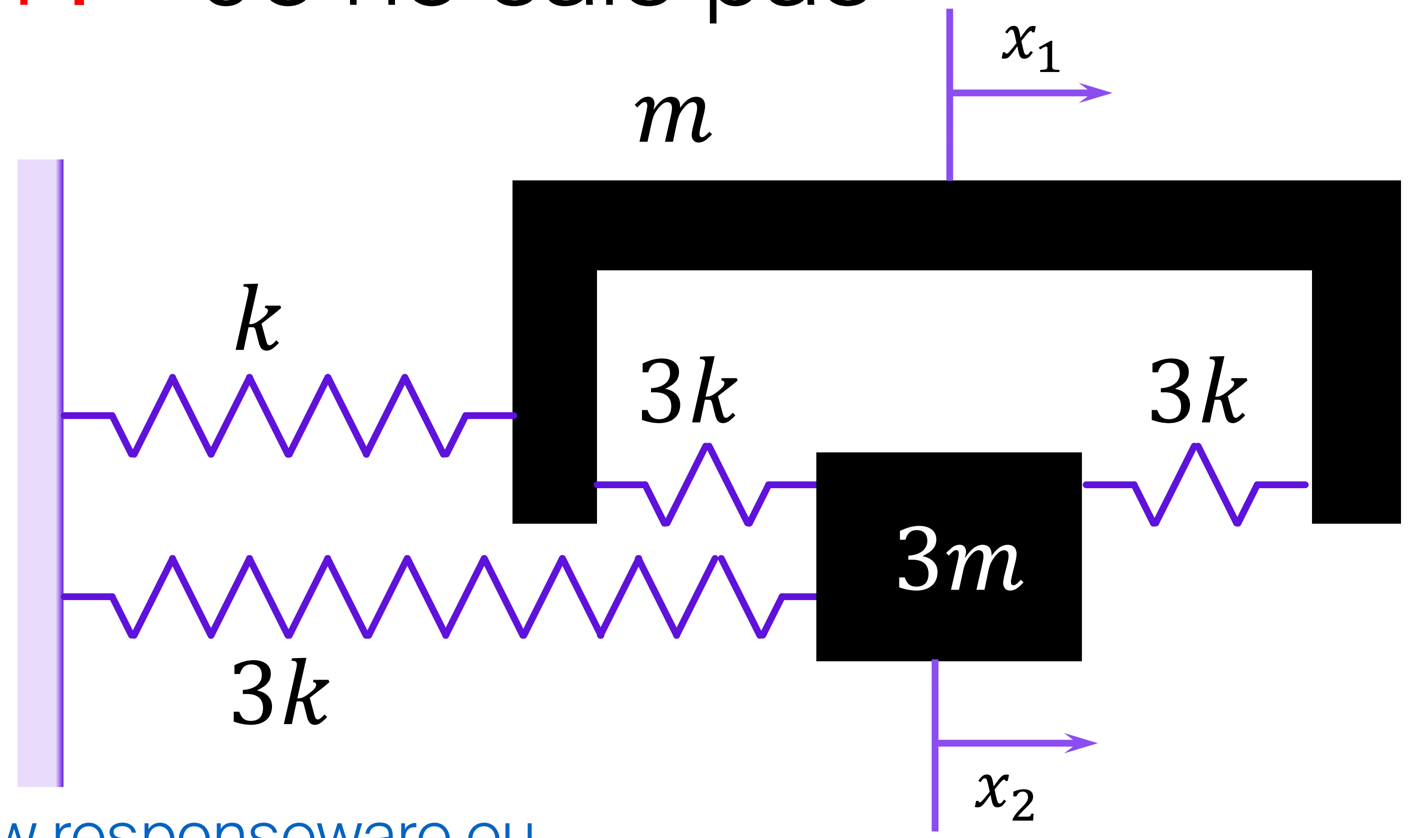
C. $\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$

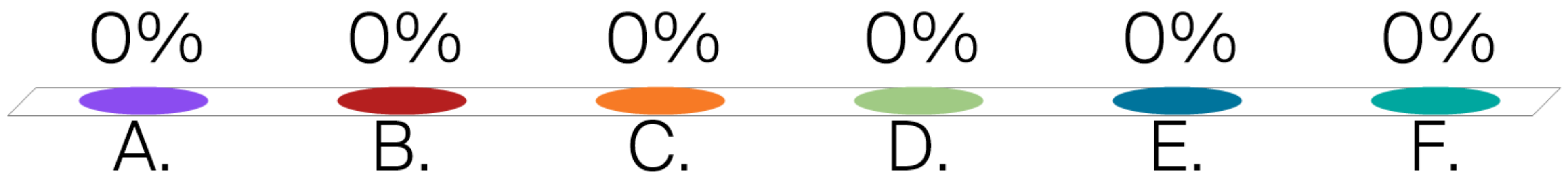
D. $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

E. Ça dépend

F. Je ne sais pas

$M^{o} = B^T M B$





Matrice de ~~masse~~ rigidité dans coordonnées q_i?

A. $\begin{pmatrix} 7k & -6k \\ -6k & 9k \end{pmatrix}$

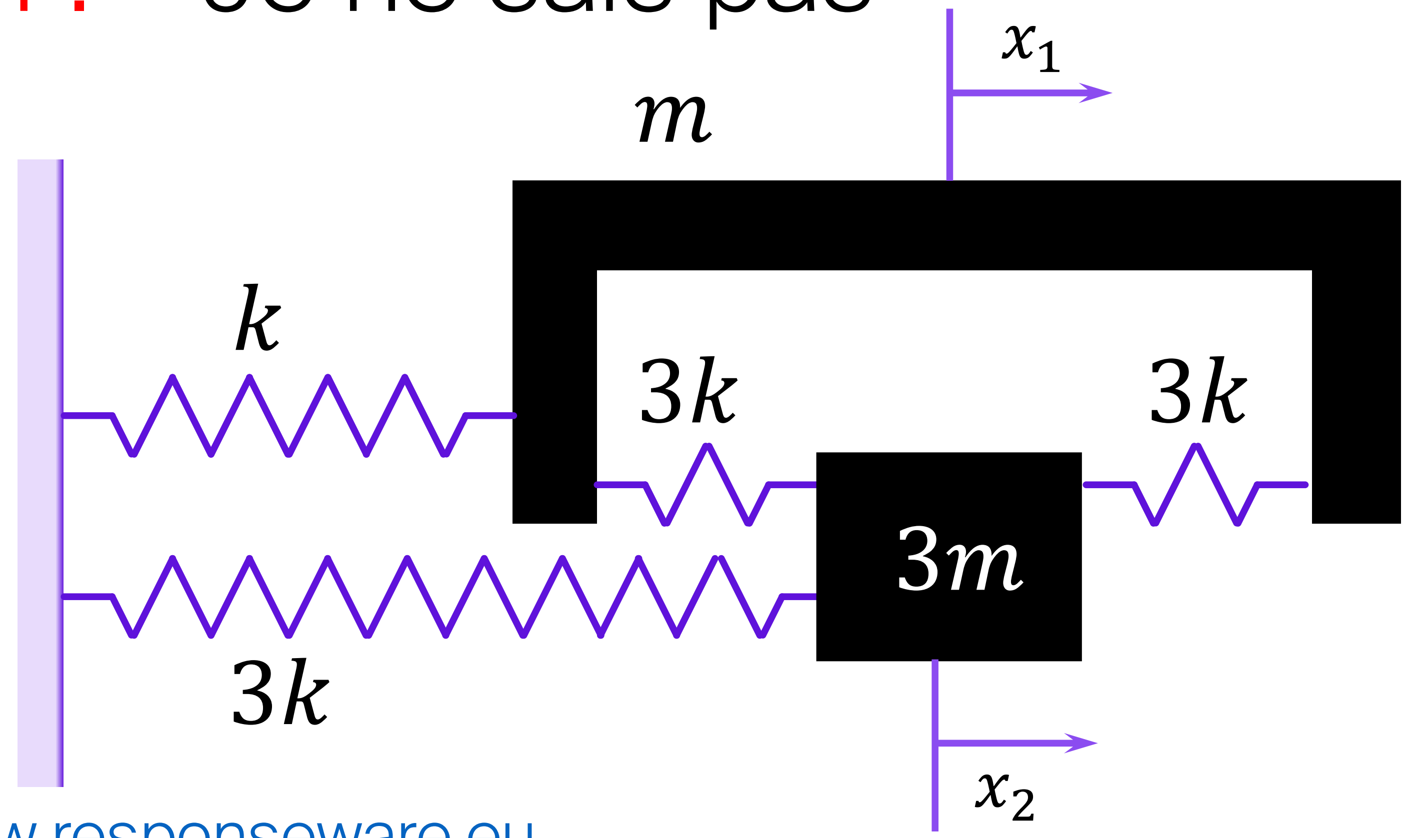
B. $\begin{pmatrix} 7 & -6 \\ -6 & 9 \end{pmatrix}$

C. $\begin{pmatrix} 7 & 6 \\ 6 & 9 \end{pmatrix} k$

D. $\begin{pmatrix} 7k & 6k \\ 6k & 9k \end{pmatrix}$

E. Ça dépend

F. Je ne sais pas



0%	0%	0%	0%	0%	0%
A.	B.	C.	D.	E.	F.

Mécanique Vibratoire - ME332 - SGM Ba5 - G. Villanueva

Semaine 14
QCMs

ME-332 – Mécanique Vibratoire

École
polytechnique
fédérale
de Lausanne

Before we start… let's connect!

- Smartphones
  - iPhone
  - Android

Download and install the app "*PointSolutions*"

pointsolutions
powered by echo360

- Computers (or Windows Phones)
  - Go to www.responseware.eu

- Choose **EUROPE Server** (if asked)
- Join session – **ME332**
- When asked – **do not input your name. Enter as anonymous.**

Final Exam

- Date: 28 Janvier (2025)
- Heure: 15h30 – 18h00 (2h30)
  - Doors open at 15h15
- Salles: CE16, PO01
- Contenu: Tout le semestre
- Formulaire:
  - Une page A4 «recto-verso»
- Pas oublier:
  - Stylo (Ni rouge ni vert – PAS crayon)
  - Calculatrice (pas matrices)
  - EPFL ID – Camipro

- Exam:
  - 1-2 questions courtes
  - 2-3 problèmes
- 70% de la note finale
- Séance de doutes:
  - Salle: MED 0 1418
  - Date: 24 Janvier 2024
  - Heure: 13h30-16h30

Matrice noyau dans coordonnées x_i?

$A = M^{-1}K$

$\boxed{M = m\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}}$

$M^{-1} = \frac{1}{m}\begin{pmatrix}1 & 0\\ 0 & 1/3\end{pmatrix}$

$K = k\begin{pmatrix}7 & -6\\ -6 & 9\end{pmatrix}$

A. $\frac{k}{m}\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$

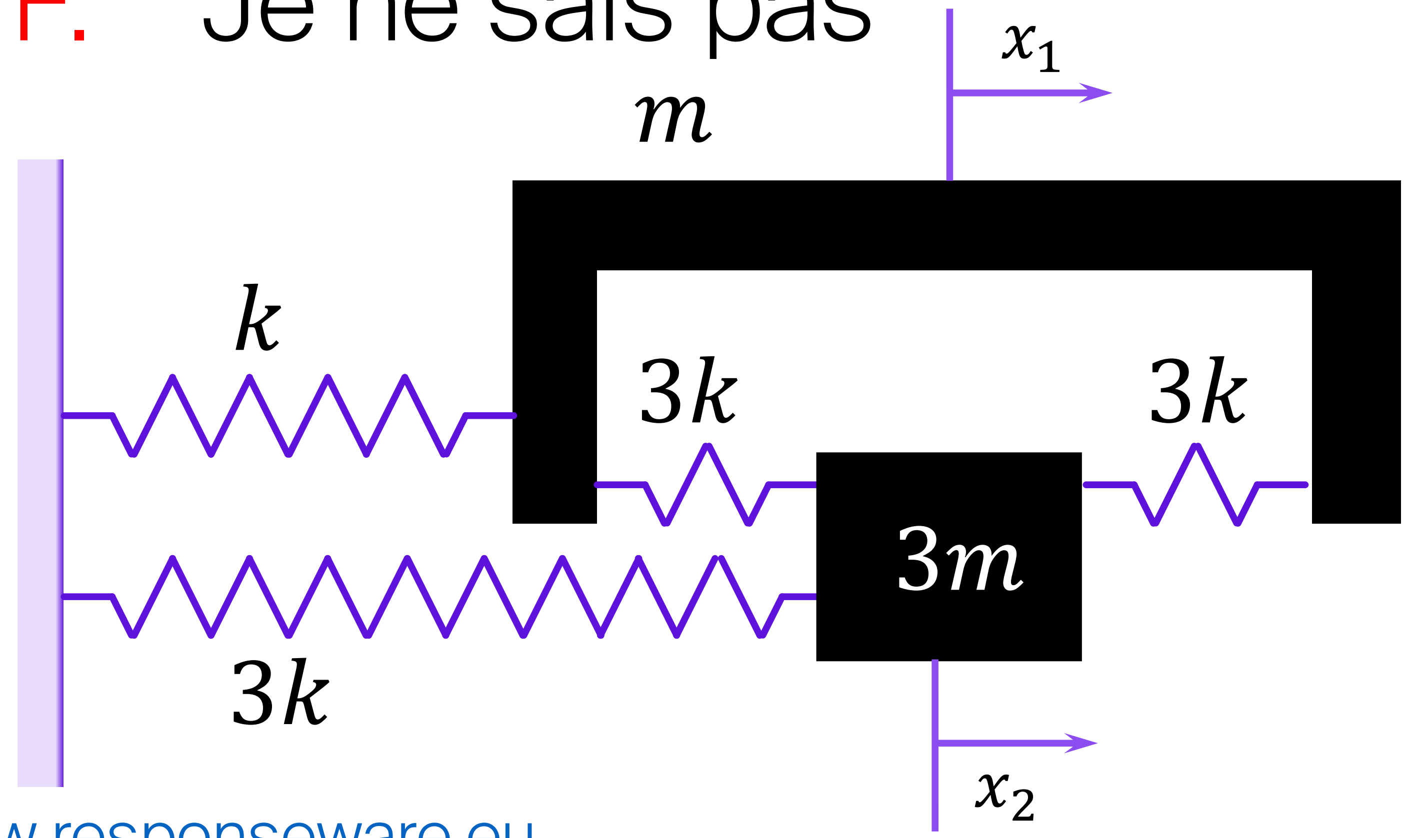
B. $\boxed{\frac{k}{m}\begin{pmatrix}7 & -6\\ -2 & 3\end{pmatrix}}$

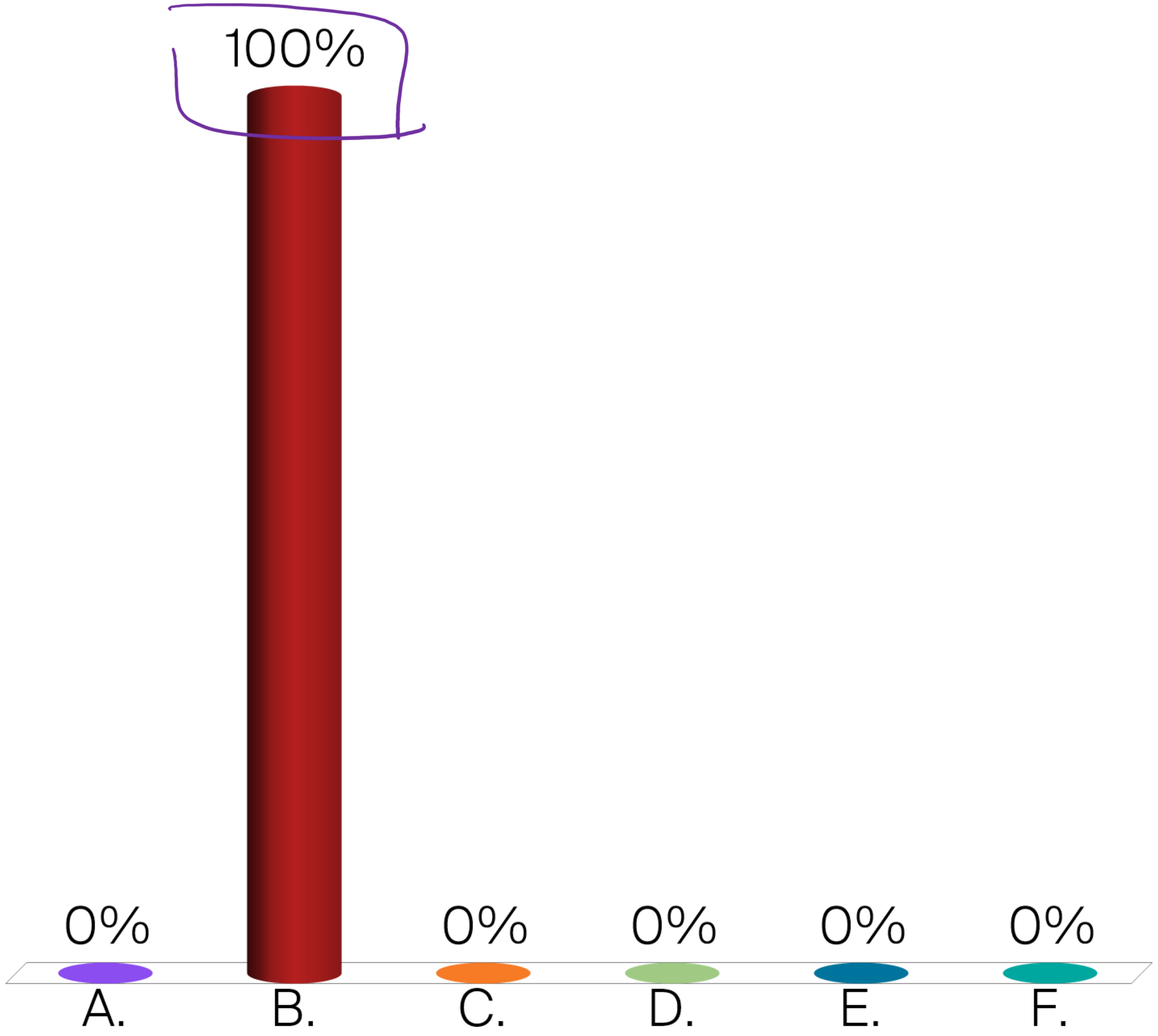
C. $\begin{pmatrix}7 & -6\\ -2 & 3\end{pmatrix}$

D. $\frac{k}{m}\begin{pmatrix}7 & -6\\ -6 & 9\end{pmatrix}$

E. Ça dépend

F. Je ne sais pas





Matrice noyau dans coordonnées q_i?

A. $\frac{k}{m}\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$

B. $\cancel{\frac{k}{m}\begin{pmatrix}7 & -6\\ -2 & 3\end{pmatrix}}$

C. $\boxed{\frac{k}{m}\begin{pmatrix}1 & 0\\ 0 & 9\end{pmatrix}}$

D. $\cancel{\frac{k}{m}\begin{pmatrix}9 & 0\\ 0 & 1\end{pmatrix}}$

E. ~~Ça dépend~~

F. Je ne sais pas

$\Delta = R^{-1} \cdot K^0 = \begin{pmatrix}\omega_1^2 & & 0\\ & \ddots & \\ 0 & & \omega_N^2\end{pmatrix}$

$A = \frac{k}{m}\begin{pmatrix}7 & -6\\ -2 & 3\end{pmatrix}$

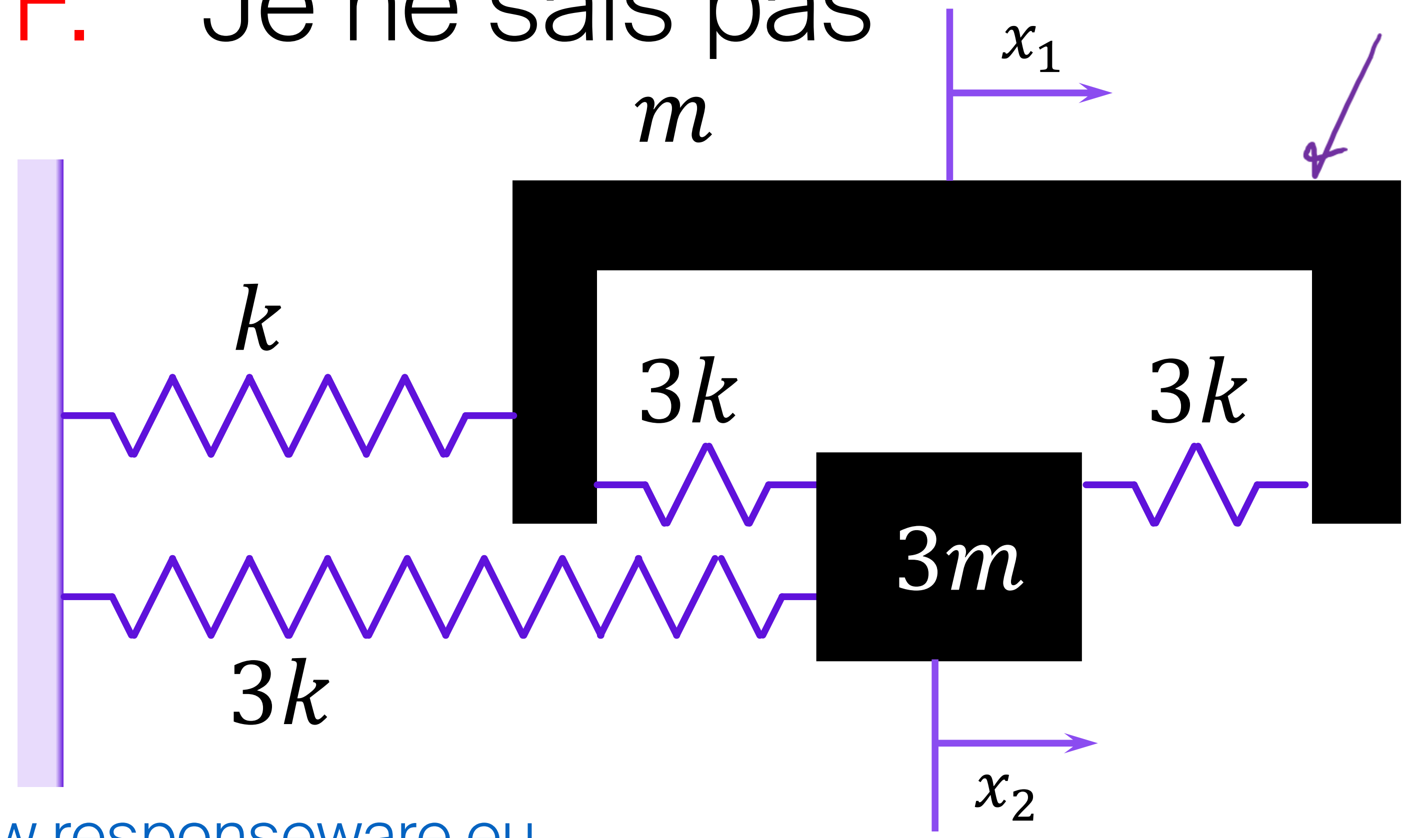
$\det(A - \omega_i^2 I) = 0$

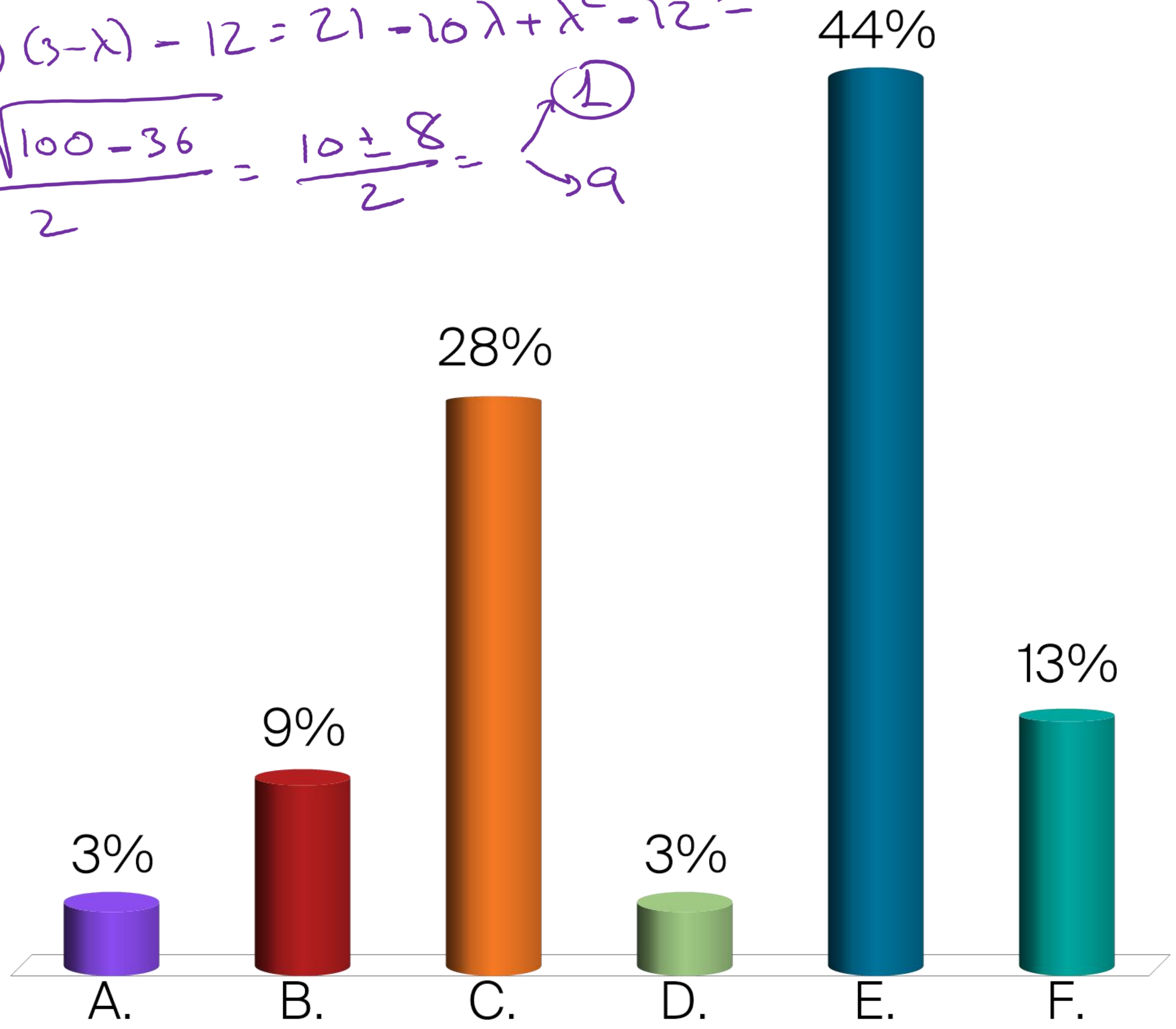
$\begin{vmatrix}7-\lambda & -6\\ -2 & 3-\lambda\end{vmatrix} = 0 = (7-\lambda)(3-\lambda) - 12 = 21 - 10\lambda + \lambda^2 - 12 = \lambda^2 - 10\lambda + 9 = 0$

$\lambda = \frac{10 \pm \sqrt{100-36}}{2} = \frac{10 \pm 8}{2} = \begin{cases}1\\ 9\end{cases}$

$\omega_I^2 = 1 \cdot \frac{k}{m}$

$\omega_{II}^2 = 9\,\frac{k}{m}$





Calculer les vecteurs propres

$$A = \frac{k}{m}\begin{pmatrix} 7 & -6 \\ -2 & 3 \end{pmatrix} \qquad \omega_I^2 = \frac{k}{m} \;;\; \omega_{II}^2 = 9\frac{k}{m}$$

$$(A - \omega_I^2 \mathbb{I})\,\vec{p}_I = 0$$

$$A\vec{\beta}_I = \omega_I^2 \vec{\beta}_I$$

$$\text{I} \rightarrow \frac{k}{m}\begin{pmatrix} 7 & -6 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ a_I \end{pmatrix} = \frac{k}{m}\begin{pmatrix} 1 \\ a_I \end{pmatrix}$$

$$\hookrightarrow 7 - 6a_I = 1 \Rightarrow a_I = 1 \Rightarrow \vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{II} \rightarrow \frac{k}{m}\begin{pmatrix} 7 & -6 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ a_{II} \end{pmatrix} = 9\frac{k}{m}\begin{pmatrix} 1 \\ a_{II} \end{pmatrix}$$

$$7 - 6a_{II} = 9 \Rightarrow a_{II} = -\frac{1}{3} \Rightarrow \vec{\beta}_{II} = \begin{pmatrix} 1 \\ -1/3 \end{pmatrix}$$

$$\vec{\beta}_I = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \qquad \vec{\beta}_{II} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\vec{\beta}_I \cdot \vec{\beta}_{II} = 1 - 1/3 = \frac{2}{3}$$

$$\vec{\beta}_I \cdot M\,\vec{\beta}_{II} = (1 \;\; 1)\cdot m\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -1/3 \end{pmatrix} = m\,(1 \;\; 1)\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$$

$$\vec{\beta}_I \cdot K \cdot \vec{\beta}_{II} = (1 \;\; 1)\,k\begin{pmatrix} 7 & -6 \\ -6 & 9 \end{pmatrix}\begin{pmatrix} 1 \\ -1/3 \end{pmatrix} = k\,(1 \;\; 1)\begin{pmatrix} 9 \\ -9 \end{pmatrix} = 0$$

Si $\vec{x}(0)=\begin{pmatrix}1\\0\end{pmatrix}$ m, $\dot{\vec{x}}(0)=\begin{pmatrix}0\\0\end{pmatrix}$, calculer $\vec{x}(t)$

$$\vec{x}(0) = \begin{pmatrix}1\\0\end{pmatrix} m = q_I(0)\cdot\vec{\beta}_I + q_{II}(0)\cdot\vec{\beta}_{II}$$

$$\dot{\vec{x}}(0) = \dot{q}_I(0)\cdot\vec{\beta}_I + \dot{q}_{II}(0)\cdot\vec{\beta}_{II} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\dot{q}_I(0) = \dot{q}_{II}(0) = 0$$

$$\begin{pmatrix}1\\0\end{pmatrix} m = \begin{pmatrix} q_I(0)+q_{II}(0)\\ q_I(0) - \frac{q_{II}(0)}{3}\end{pmatrix} \Rightarrow q_I(0) = \frac{q_{II}(0)}{3}$$

$$q_{II} = 3q_I(0) \Rightarrow q_I(0) + 3q_I(0) = 1\,m \Rightarrow \boxed{q_I(0) = \frac{1}{4}\,m;\ q_{II}(0) = \frac{3}{4}\,m}$$

$$q_I(0) \Rightarrow q_I(t) = \frac{1}{4}\cos(\omega_I t)\ m$$

$$q_{II}(0) \Rightarrow q_{II}(t) = \underbrace{Q_{II}}_{q_{II}(0)}\cdot\cos(\omega_{II} t - \cancel{\varphi_{II}}) = \frac{3}{4}\cdot\cos(\omega_{II} t)\ m$$

$\varphi_{II} = 0$

$$\vec{x}(t) = B\cdot\vec{q}(t) = q_I(t)\cdot\vec{\beta}_I + q_{II}(t)\cdot\vec{\beta}_{II} = \begin{pmatrix}\frac{1}{4}\cos(\omega_I t) + \frac{3}{4}\cos(\omega_{II} t)\\ \frac{1}{4}\cos(\omega_I t) - \frac{1}{4}\cos(\omega_{II} t)\end{pmatrix} m$$

$$\vec{\beta}_I = \begin{pmatrix}1\\1\end{pmatrix} \qquad \vec{\beta}_{II} = \begin{pmatrix}1\\-1/3\end{pmatrix}$$

Matrice de dissipation dans coordonnées x_i?

A. $\begin{pmatrix} c & 0 \\ 0 & 3c \end{pmatrix}$

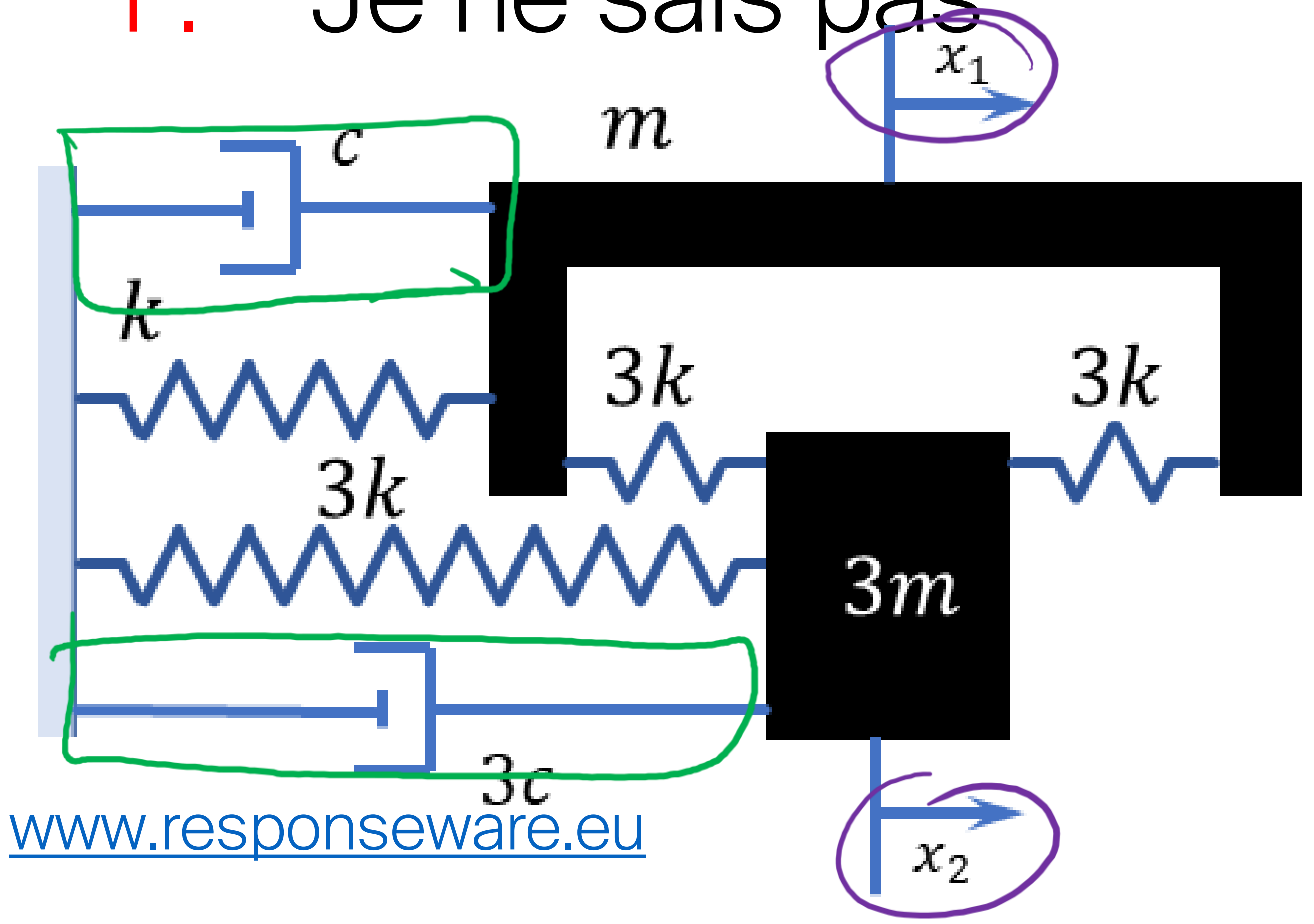
B. $\begin{pmatrix} 3c & 0 \\ 0 & c \end{pmatrix}$

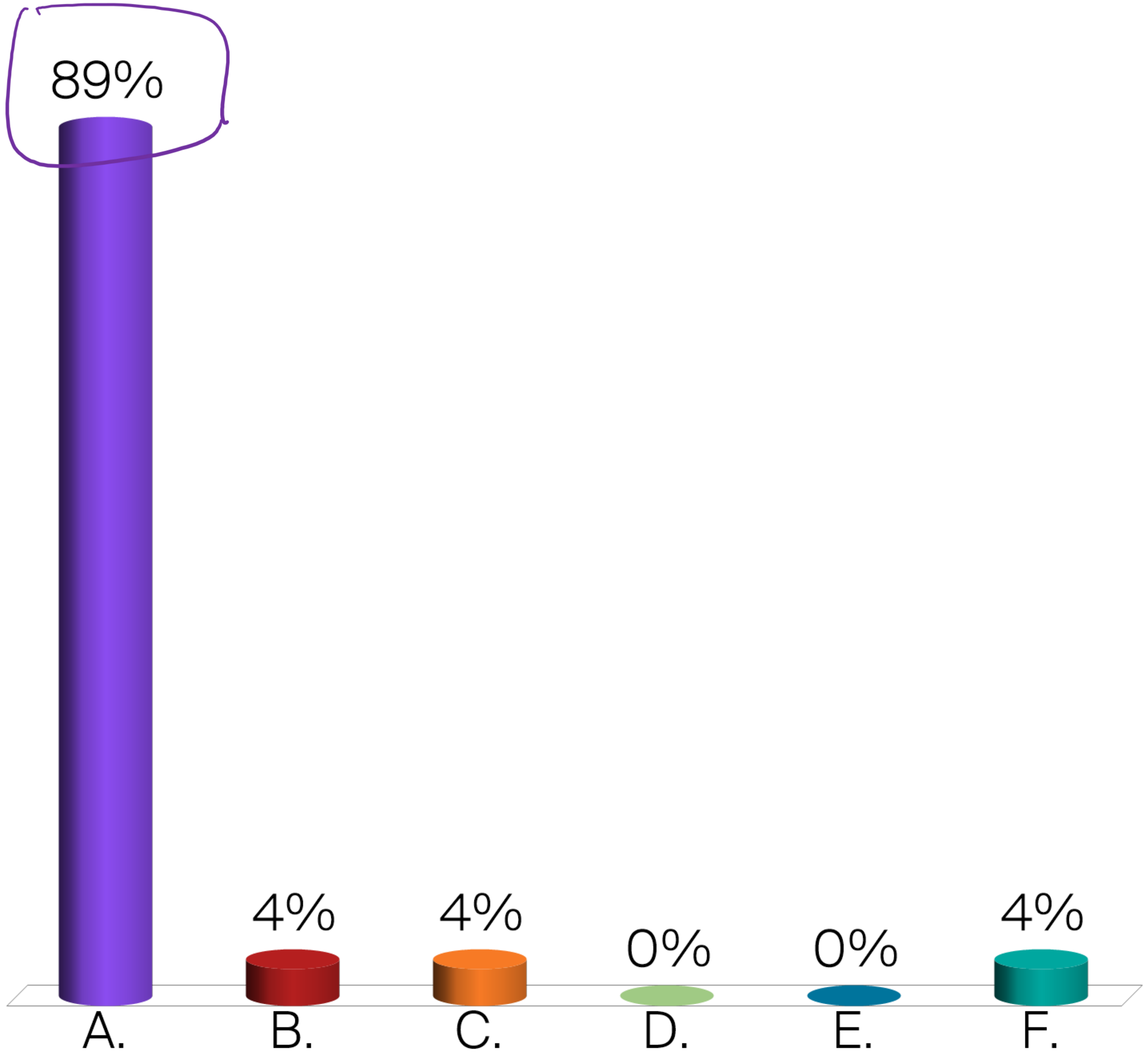
C. $\begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix}$

D. $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

E. Ça dépend

F. Je ne sais pas





Système Caughey?

A. Oui

B. Non

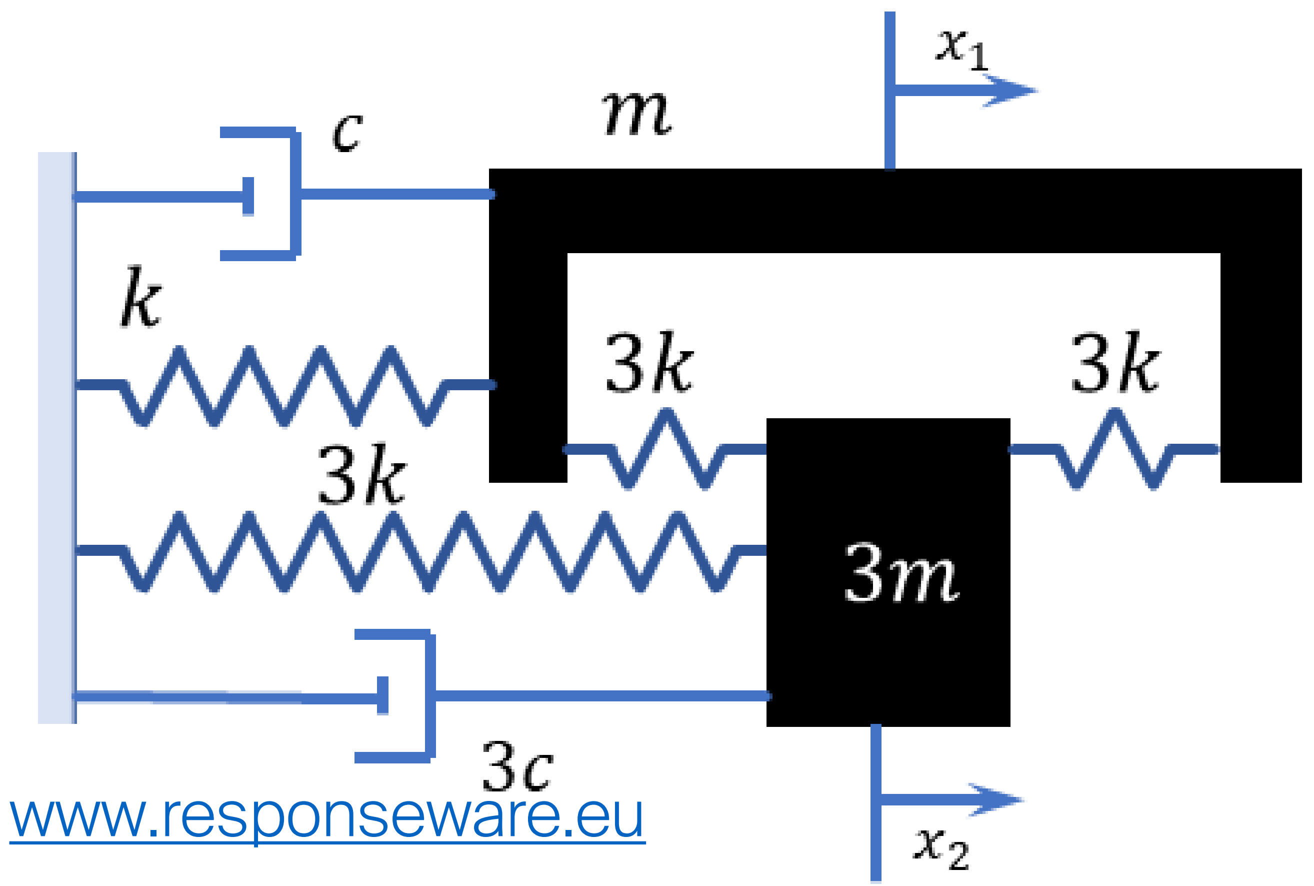
C. Je ne sais pas

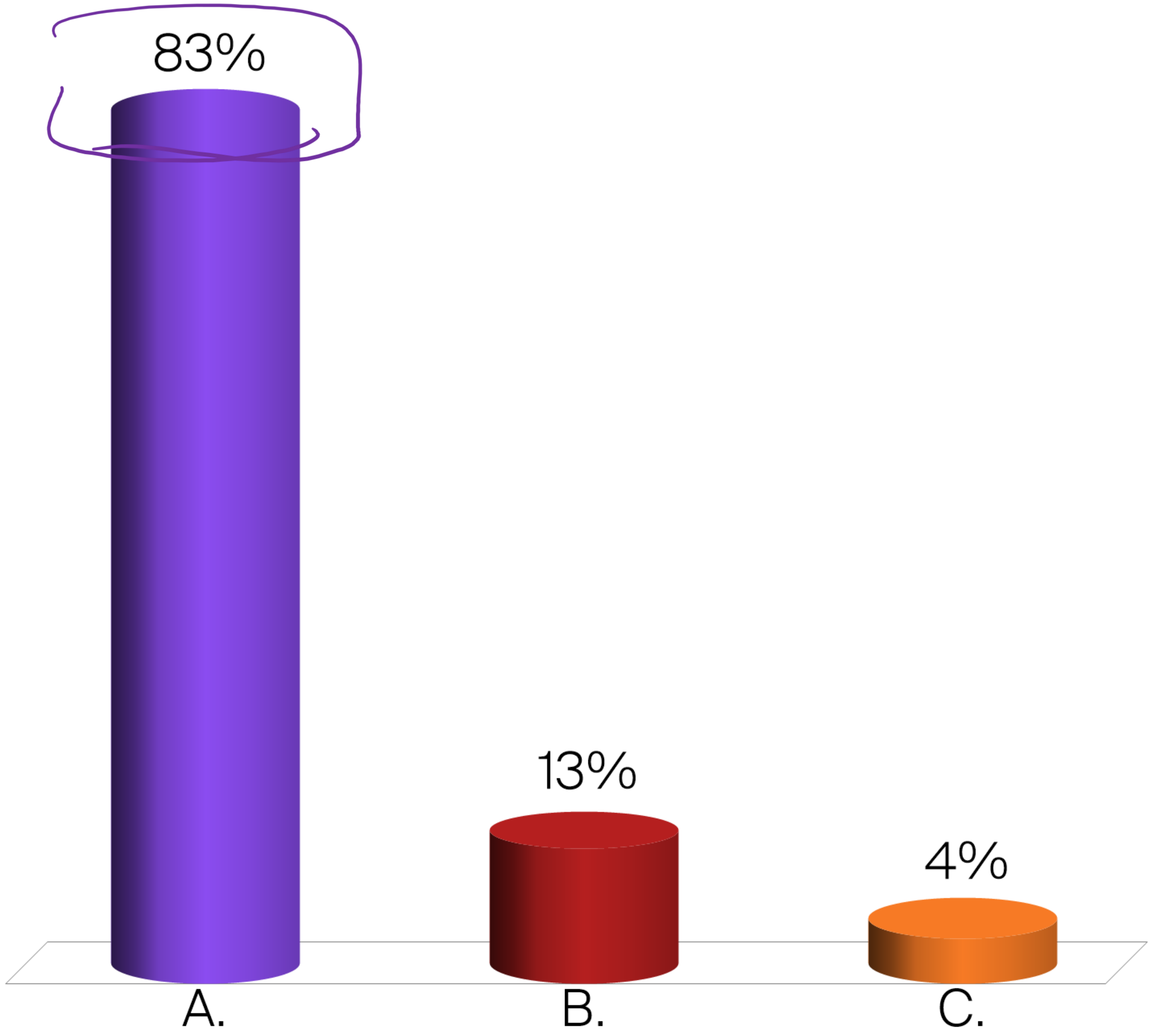
$C = c\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$ $M = m\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$ $K = k\begin{pmatrix}7 & -6\\ -6 & 9\end{pmatrix}$

$C \cdot M^{-1} K = K \cdot M^{-1} \cdot C$

$C = \alpha M + \beta K$

$C = \frac{c}{m} \cdot M$





Matrice de dissipation dans coordonnées q_i?

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \vec{\beta}_{II} = \begin{pmatrix} 1 \\ -1/3 \end{pmatrix}$$

A. $\begin{pmatrix} c & 0 \\ 0 & 3c \end{pmatrix}$

B. $\begin{pmatrix} 3c & 0 \\ 0 & c \end{pmatrix}$

C. $\begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix}$

D. $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

E. Ça dépend

F. Je ne sais pas

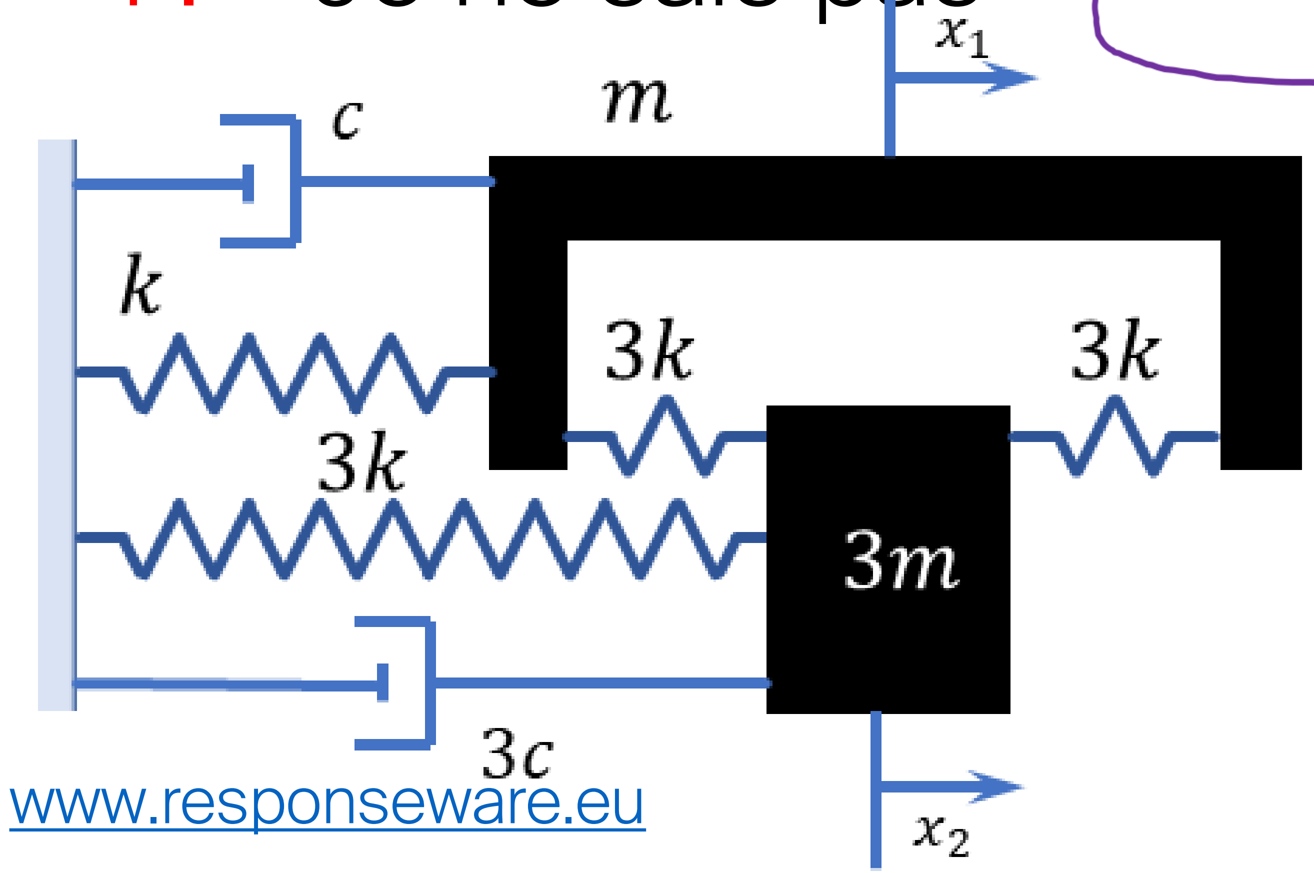
$$C^0 = B^T C B$$

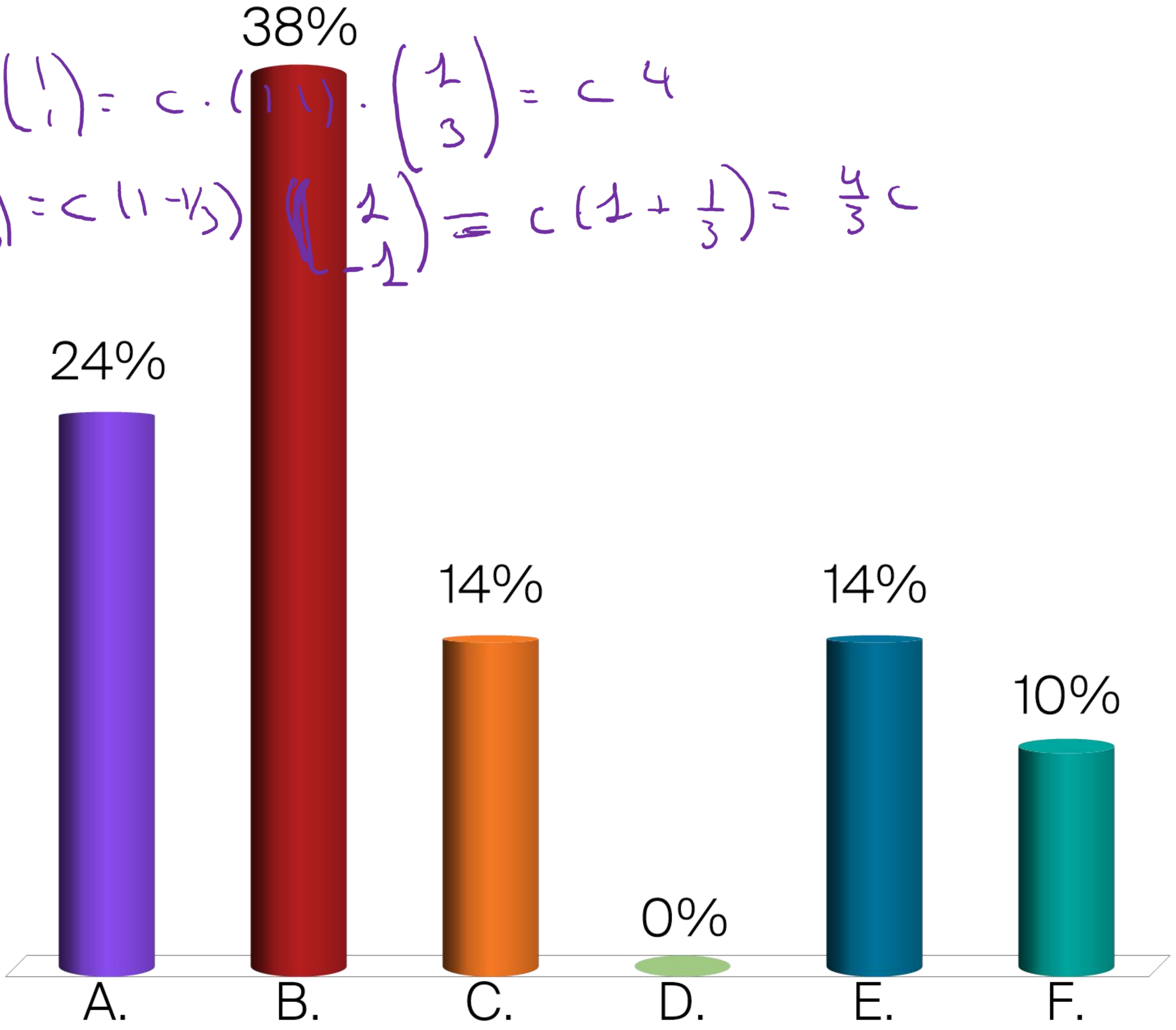
$$C^0 = \begin{pmatrix} \square & 0 \\ 0 & \square \end{pmatrix}$$

$$C^0_{1,1} = \vec{\beta}_I C \vec{\beta}_I = (1\ 1) \cdot c \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = c \cdot (1\ 1) \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = c\,4$$

$$C^0_{2,2} = \vec{\beta}_{II} \cdot C \vec{\beta}_{II} = (1\ {-1/3})\, c \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ -1/3 \end{pmatrix} = c\,(1\ {-1/3}) \begin{pmatrix} 1 \\ -1 \end{pmatrix} = c\left(1 + \frac{1}{3}\right) = \frac{4}{3} c$$

$$\boxed{C^0 = c \begin{pmatrix} 4 & 0 \\ 0 & 4/3 \end{pmatrix}}$$

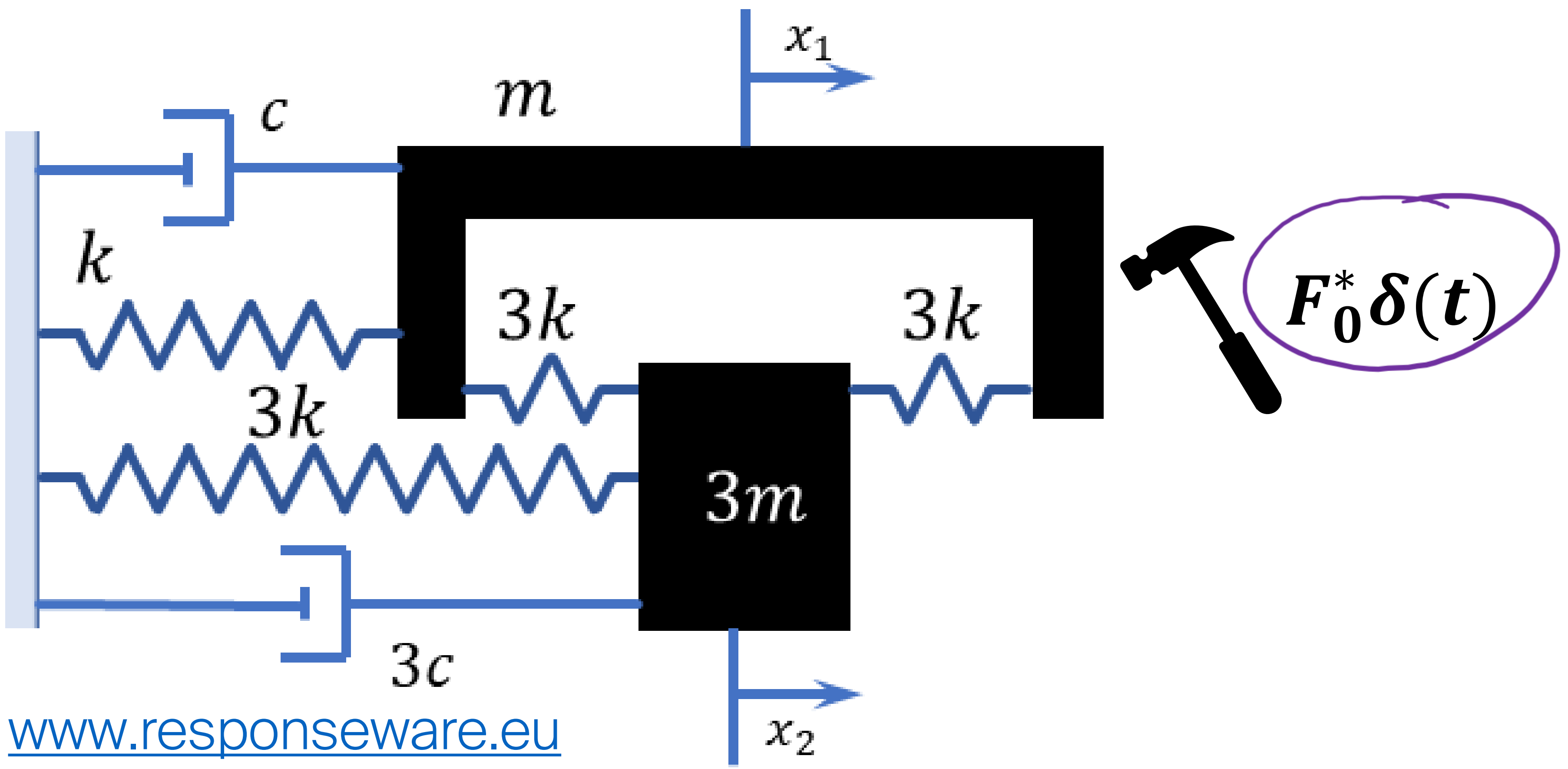




Calculer $\vec{x}(t)$ en régime permanent

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \vec{\beta}_{II} = \begin{pmatrix} 1 \\ -1/3 \end{pmatrix}$$

$\vec{x}(t) = 0$



Vecteur de force en coordonnées x_i?

A. $F_0^*\delta(t)\begin{pmatrix}1\\1\end{pmatrix}$

B. $F_0^*\delta(t)\begin{pmatrix}1\\0\end{pmatrix}$

C. $F_0^*\delta(t)\begin{pmatrix}0\\1\end{pmatrix}$

D. $F_0^*\delta(t)\begin{pmatrix}1\\-{}^{1}/_{3}\end{pmatrix}$

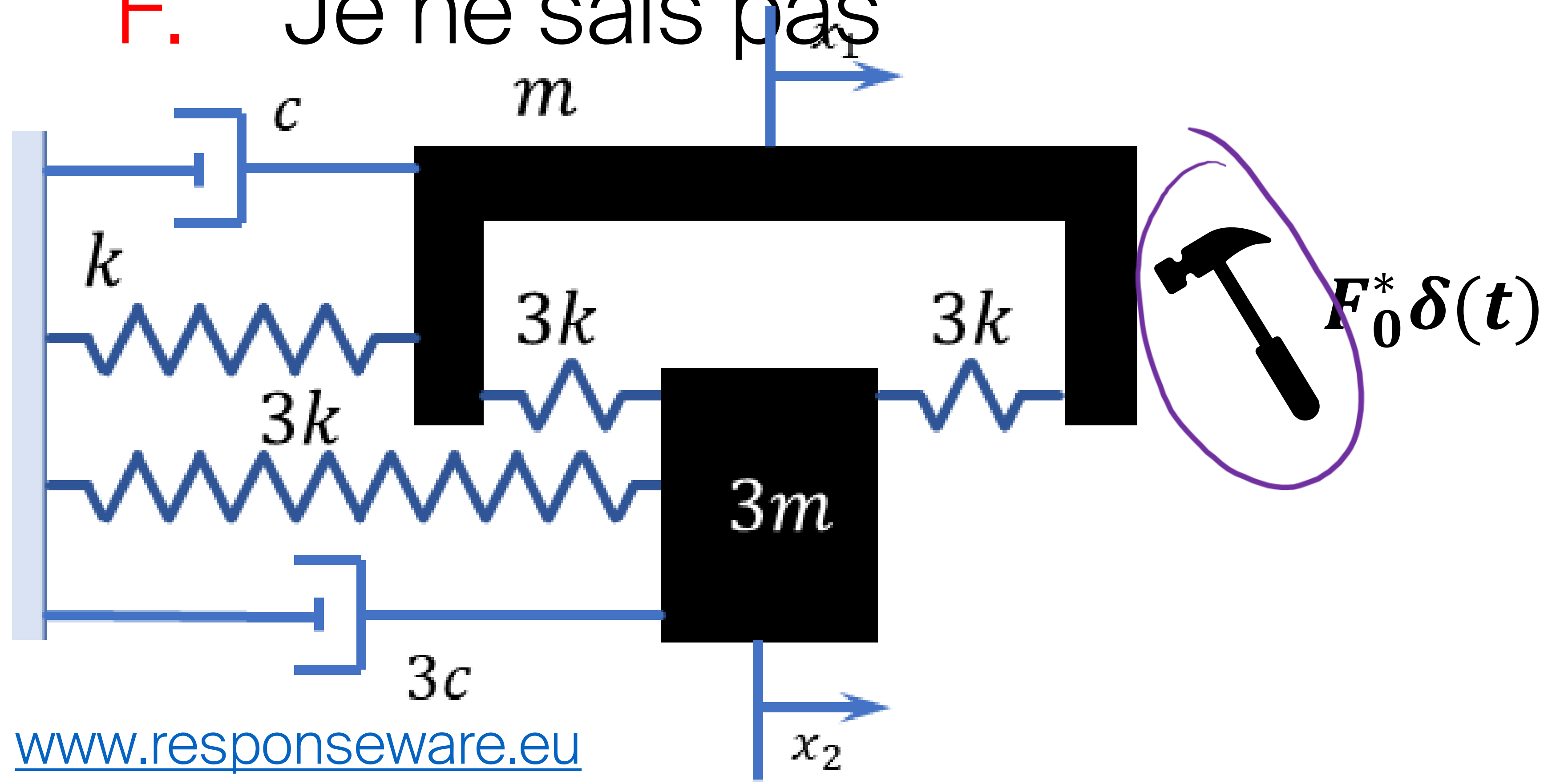
E. Ça dépend

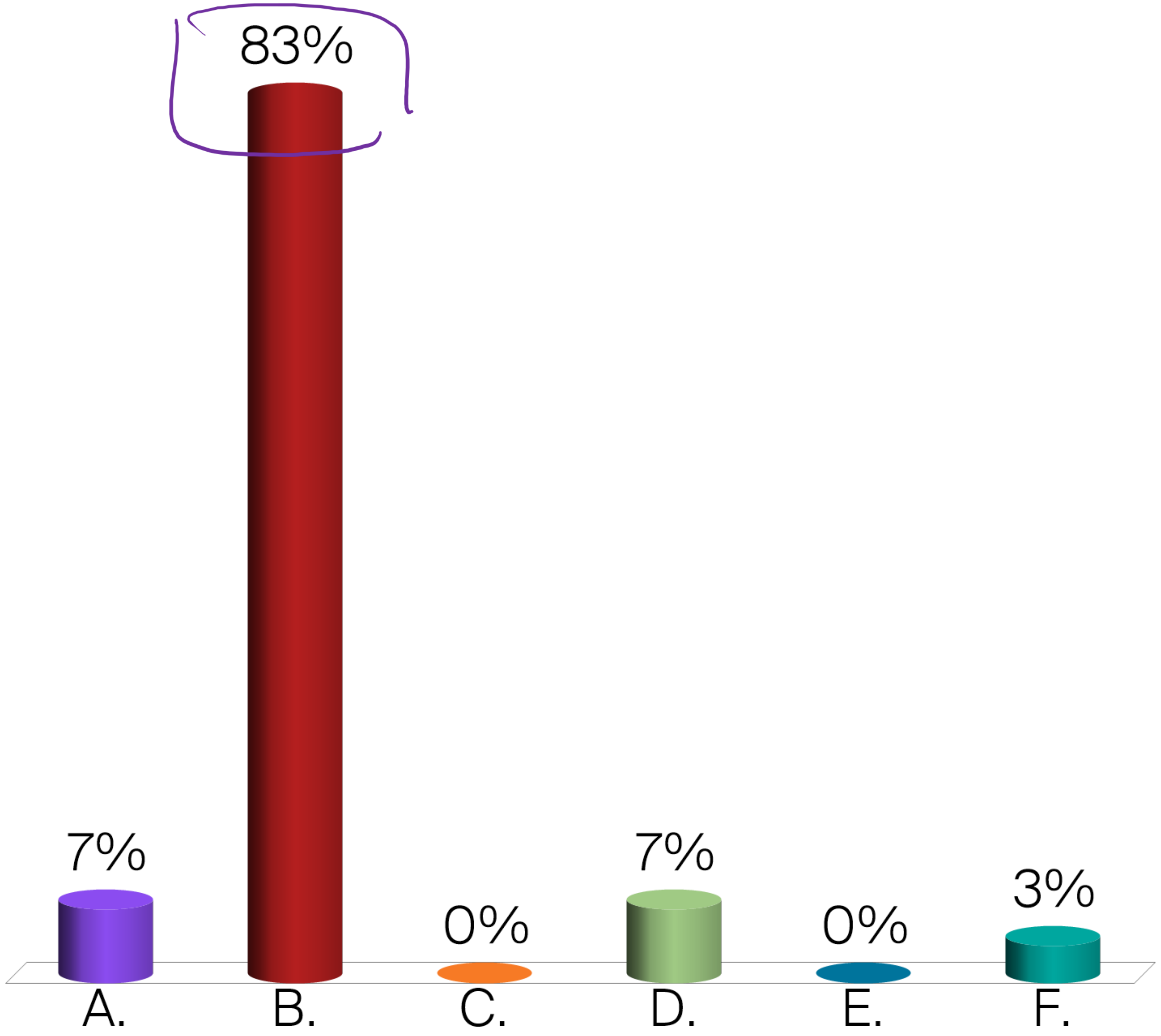
F. Je ne sais pas

$m\ddot{x}_1 + \ldots\ldots = F_0\,\delta(t)$

$m\ddot{x}_2 + \ldots\ldots = 0$

$\vec{\beta}_I = \begin{pmatrix}1\\1\end{pmatrix};\quad \vec{\beta}_{II} = \begin{pmatrix}1\\-{}^{1}/_{3}\end{pmatrix}$





Vecteur de force en coordonnées q_i?

$\vec{\beta}_I = \begin{pmatrix}1\\1\end{pmatrix}$; $\vec{\beta}_{II} = \begin{pmatrix}1\\-1/3\end{pmatrix}$

A. $F_0^*\delta(t)\begin{pmatrix}1\\1\end{pmatrix}$

B. $F_0^*\delta(t)\begin{pmatrix}1\\0\end{pmatrix}$

C. $F_0^*\delta(t)\begin{pmatrix}0\\1\end{pmatrix}$

D. $F_0^*\delta(t)\begin{pmatrix}1\\-1/3\end{pmatrix}$

E. Ça dépend

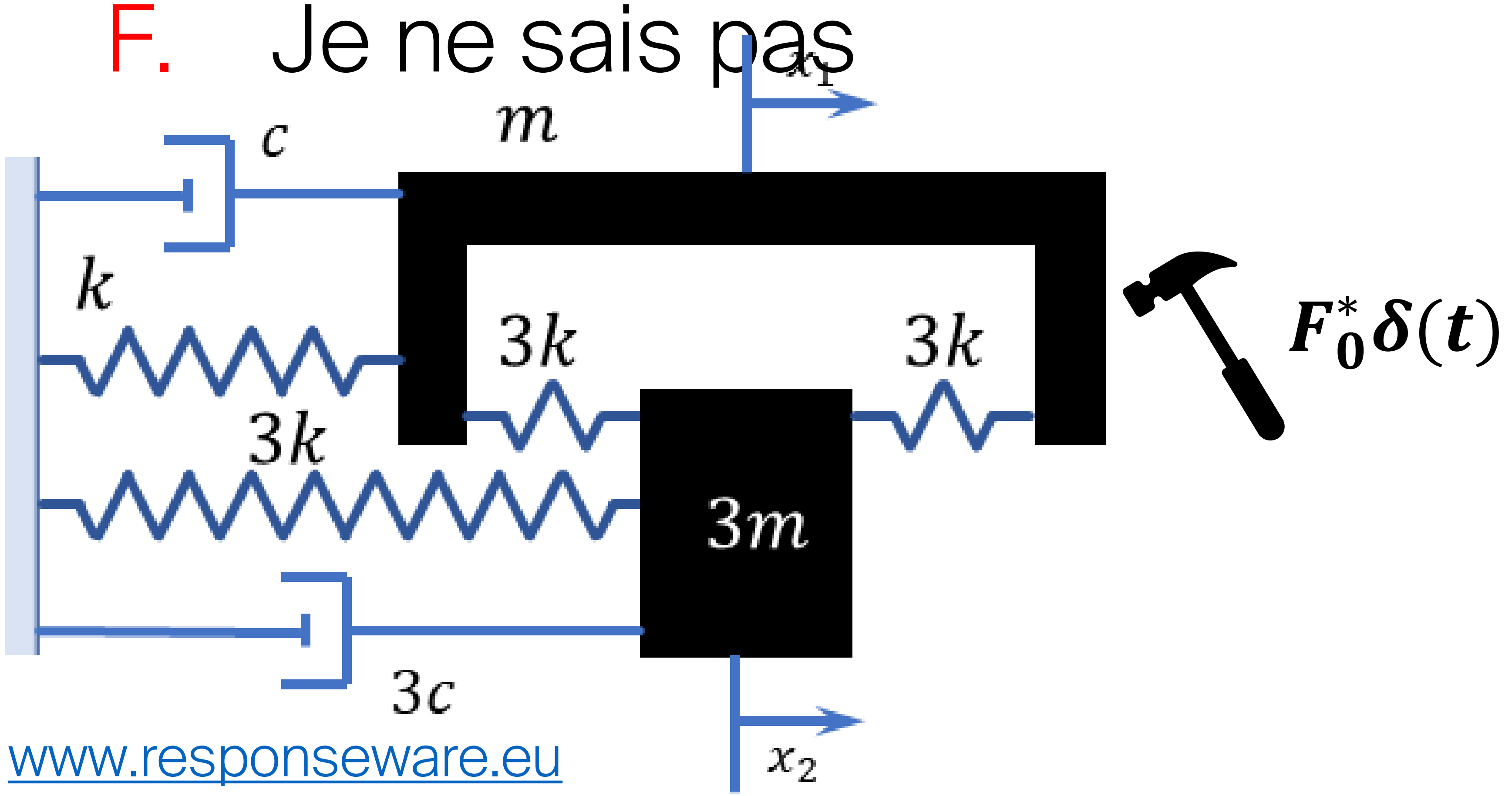
F. Je ne sais pas

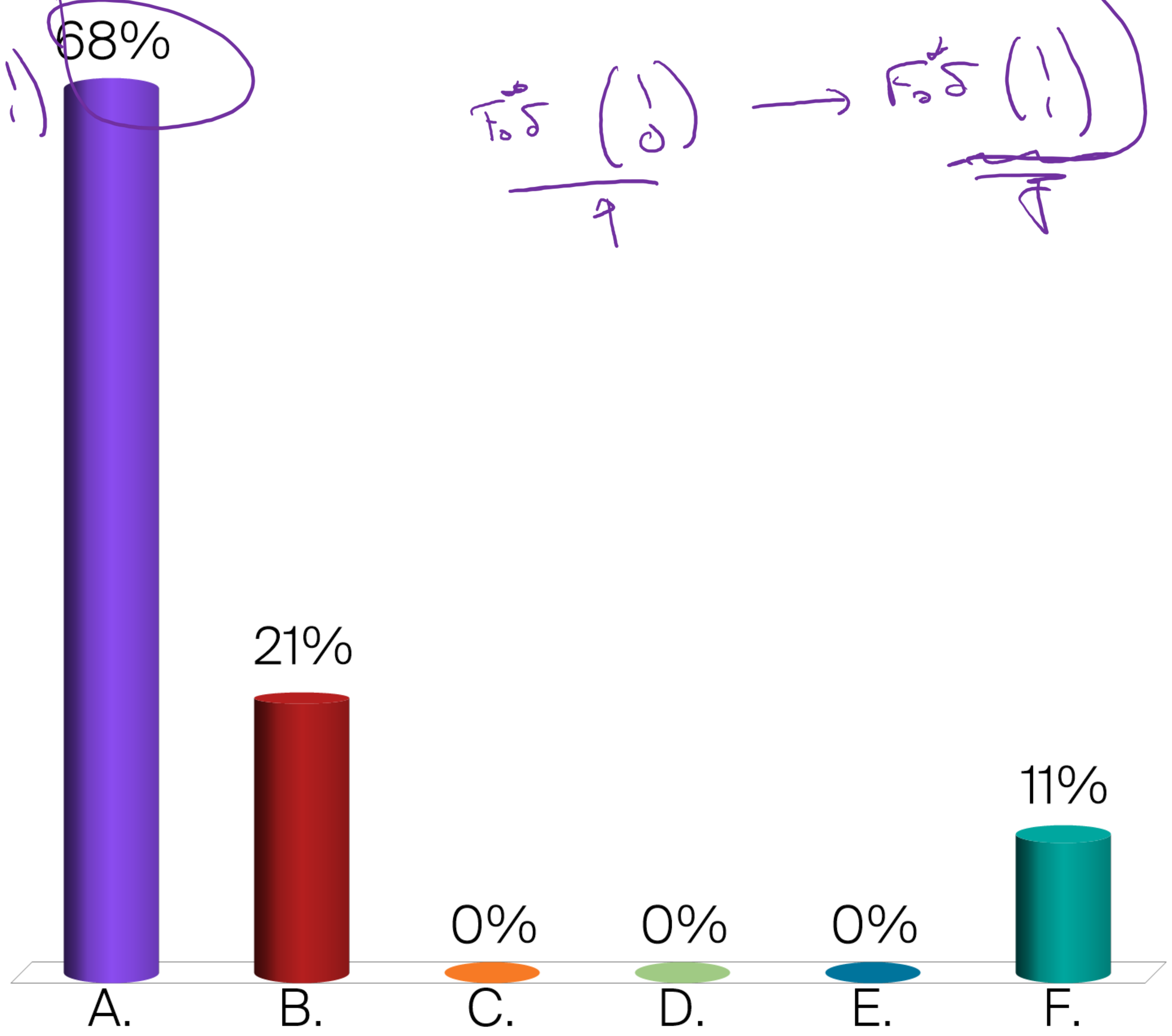
$\vec{F}_q = \beta^T \cdot \vec{F}_{0,x}$

$\begin{pmatrix}\beta_I^T\\ \beta_{II}^T\end{pmatrix} = \begin{pmatrix}1 & 1\\ 1 & -1/3\end{pmatrix}$

$\vec{F}_q = \begin{pmatrix}1 & 1\\ 1 & -1/3\end{pmatrix} F_0\delta \cdot \begin{pmatrix}1\\0\end{pmatrix} = F_0^*\delta\begin{pmatrix}1\\1\end{pmatrix}$

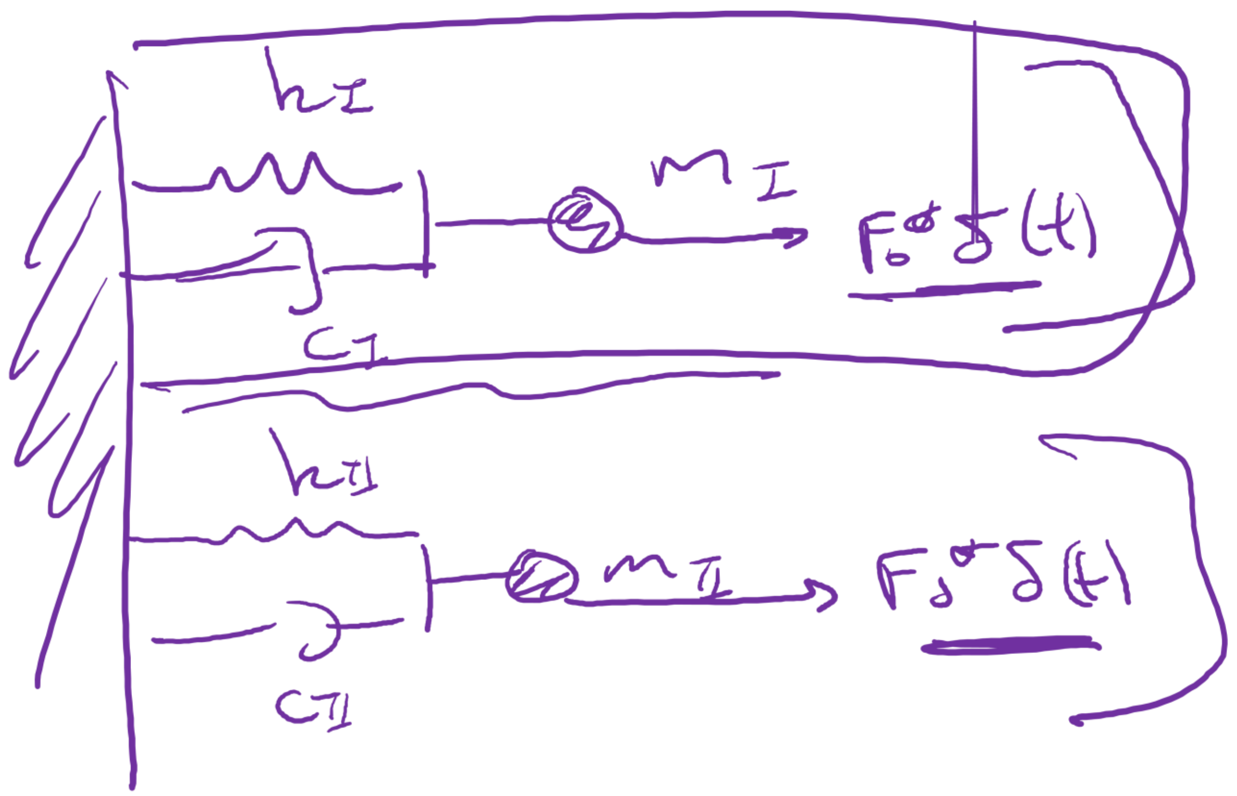
$\underbrace{F_0^*\delta\begin{pmatrix}1\\0\end{pmatrix}}_{x} \longrightarrow \underbrace{F_0^*\delta\begin{pmatrix}1\\1\end{pmatrix}}_{q}$

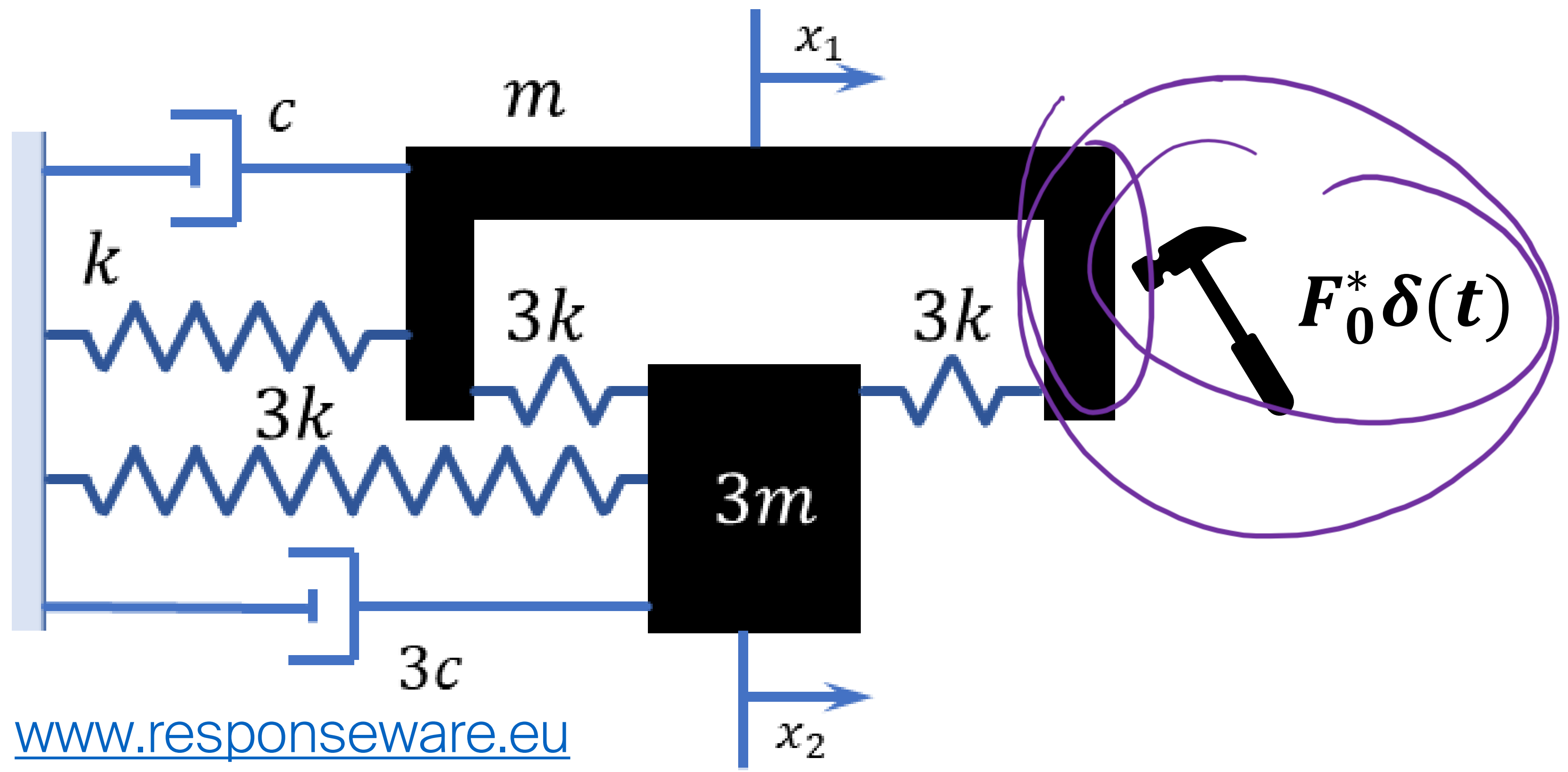




Calculer $\vec{x}(t)$

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \vec{\beta}_{II} = \begin{pmatrix} 1 \\ -1/3 \end{pmatrix}$$

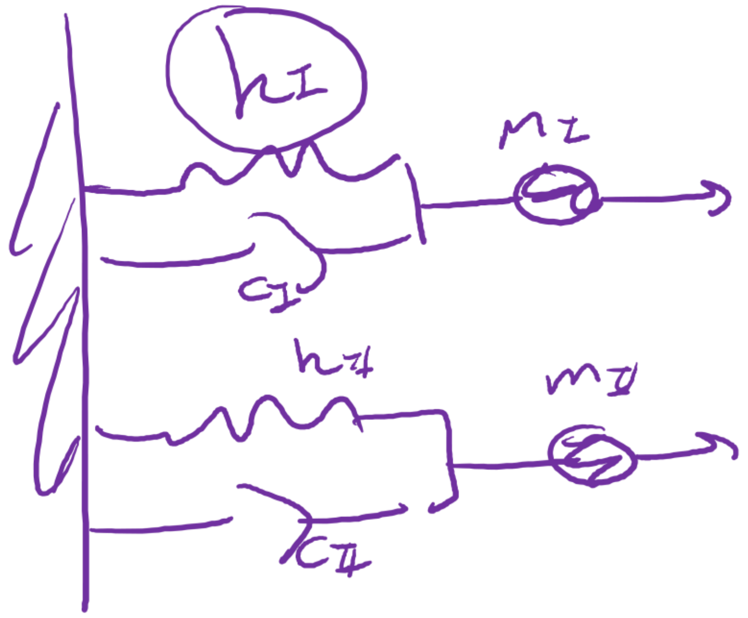




Calculer $\vec{x}(t)$ en régime permanent

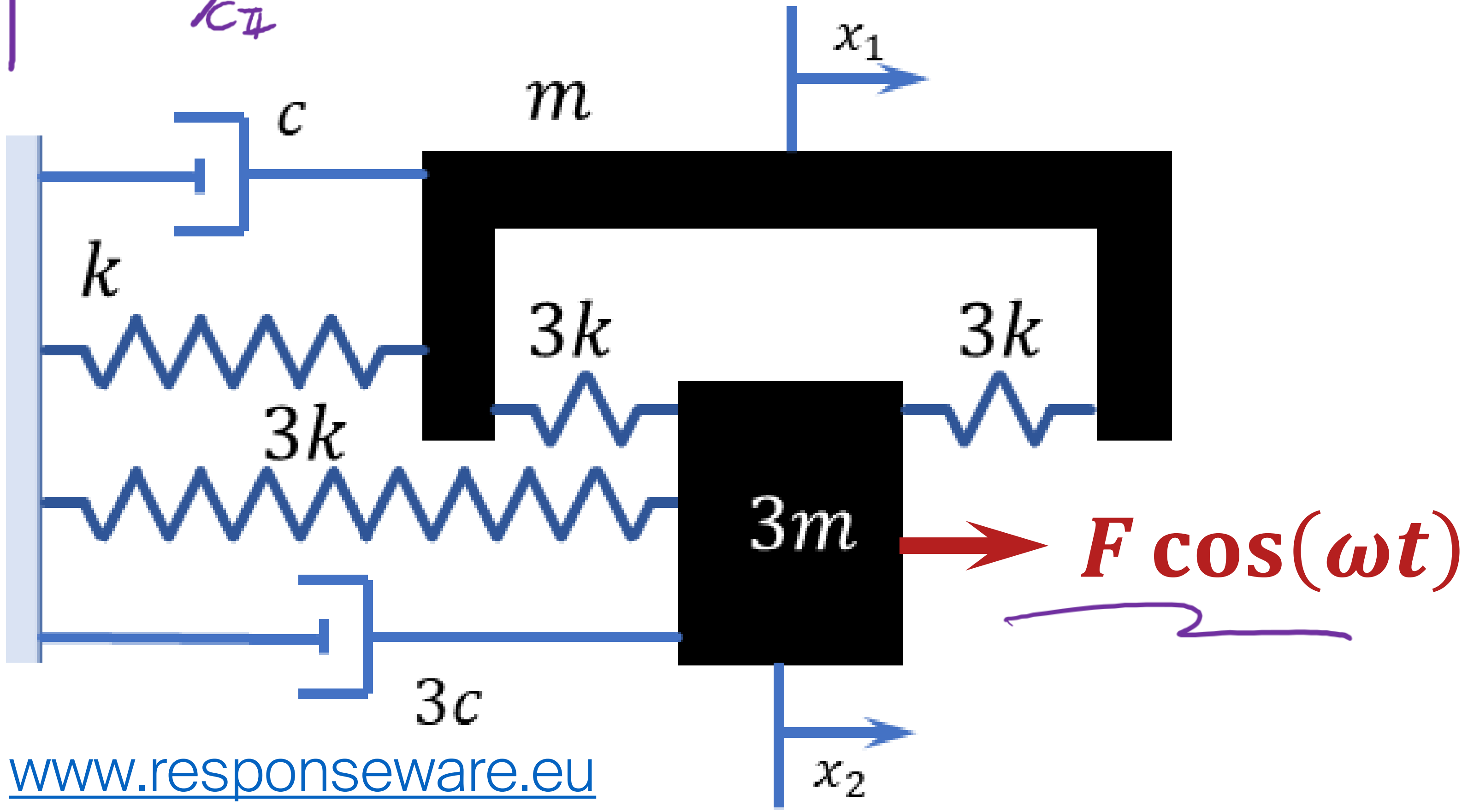
$\vec{x}(t) = B \cdot \vec{q} = q_I(t)\cdot\vec{\beta}_I + q_{II}(t)\cdot\vec{\beta}_{II}$

$\vec{\beta}_I = \begin{pmatrix}1\\1\end{pmatrix}; \quad \vec{\beta}_{II} = \begin{pmatrix}1\\-1/3\end{pmatrix}$



$F_I = F\cos\omega t \Rightarrow q_I(t) = \frac{F}{k_I}\cdot\mu_I\cdot\cos(\omega t - \varphi_I)$; $\mu_I = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_I}\right)^2\right)^2 + 4\eta_I^2\left(\frac{\omega}{\omega_I}\right)^2}}$; $\varphi_I = \frac{2\eta_I\frac{\omega}{\omega_I}}{1-\left(\frac{\omega}{\omega_I}\right)^2}$

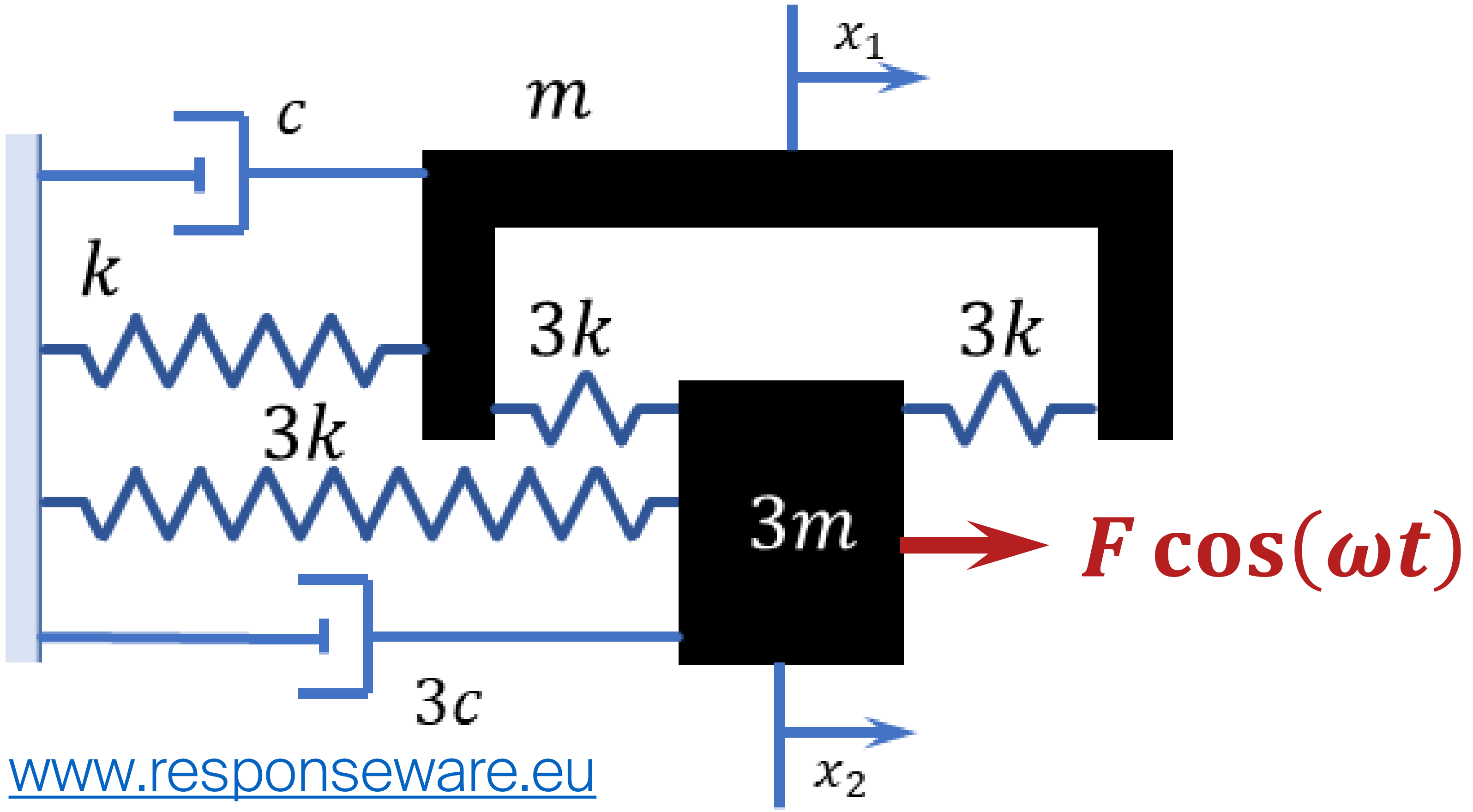
$F_{II} = -\frac{F}{3}\cos\omega t \Rightarrow q_{II}(t) = \frac{-F}{3k_{II}}\mu_{II}\cos(\omega t - \varphi_{II})$; $\mu_{II} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_{II}}\right)^2\right)^2 + 4\eta_{II}^2\left(\frac{\omega}{\omega_{II}}\right)^2}}$; $\varphi_{II} = \frac{2\eta_{II}\frac{\omega}{\omega_{II}}}{1-\left(\frac{\omega}{\omega_{II}}\right)^2}$



$\vec{F}_q = B^T\cdot\vec{F}_x = \begin{pmatrix}1 & 1\\ 1 & -1/3\end{pmatrix}\begin{pmatrix}0\\ F\cos(\omega t)\end{pmatrix} = \begin{pmatrix}F\cos(\omega t)\\ -F/3\cos\omega t\end{pmatrix}$

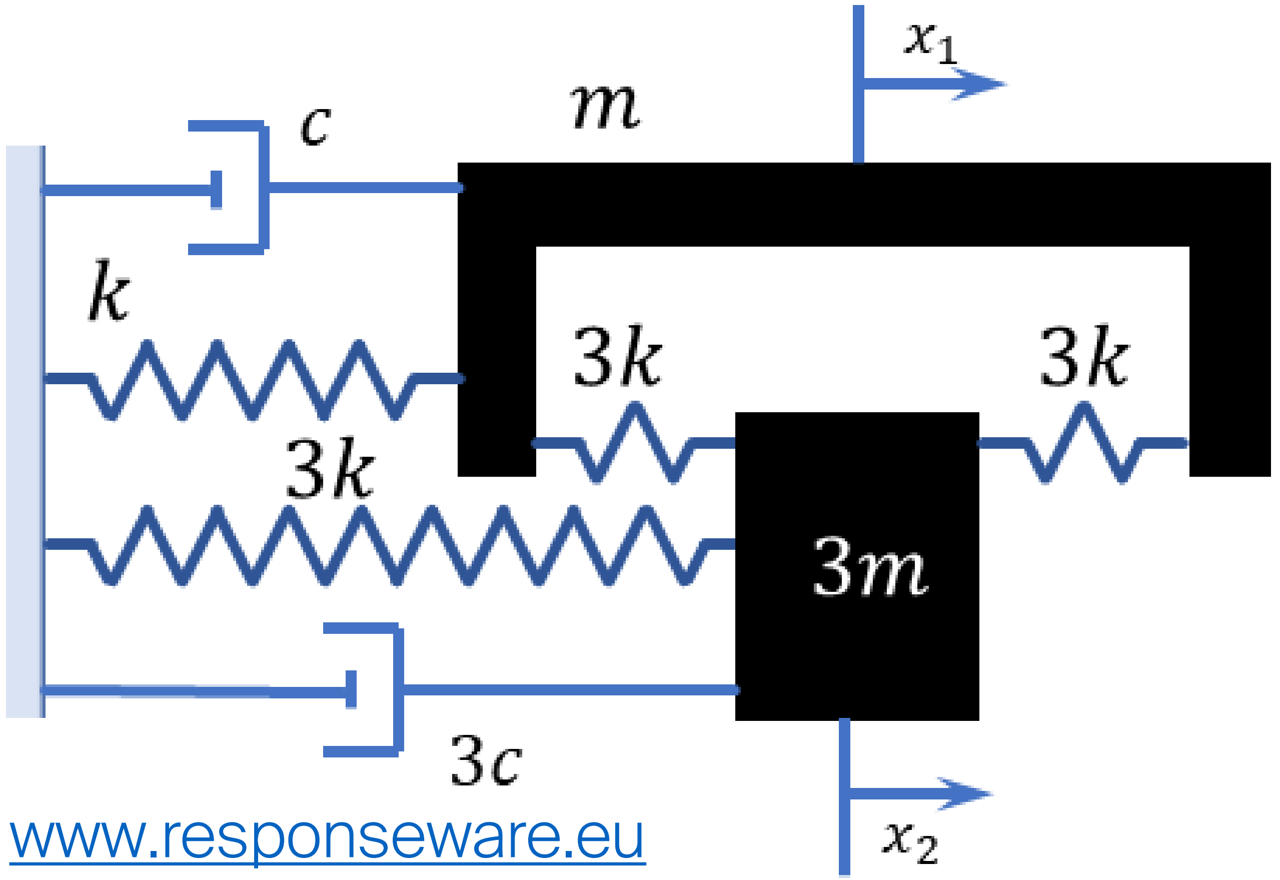
Calculer $\vec{x}(t)$ en régime permanent

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \vec{\beta}_{II} = \begin{pmatrix} 1 \\ -1/3 \end{pmatrix}$$



Fonction de Transfert (harmonique)

$$\vec{\beta}_I = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \vec{\beta}_{II} = \begin{pmatrix} 1 \\ -{}^{1}/_{3} \end{pmatrix}$$



$$\underline{\vec{X}}(j\omega) = \sum_{i=1}^{n} \vec{v}_i \underline{Q_i}(j\omega) = \left[\sum_{i=1}^{n} \frac{\vec{v}_i \otimes \vec{v}_i}{m_{0i}(-\omega^2 + 2j\eta_i\omega_{0i}\omega + \omega_{0i}{}^2)}\right] \underline{\vec{F}}(j\omega) = \underline{H}(j\omega)\underline{\vec{F}}(j\omega)$$